Adaptive Bipartite Consensus of Multi-Agent Systems with Parameter Uncertainty and Leader of Nonzero Input under Signed Digraph

Qiufu Wang and Zhanshan Wang

***Abstract*—This article investigates the bipartite consensus problem for a class of multi-agent systems (MASs) with parameter uncertainty and leader of nonzero input under a signed digraph. Due to the unknown and time-varying input signal of the leader, followers are unable to obtain accurate information about the leader. For this purpose, a distributed adaptive observer is designed based on parameterization theory. Then, considering the existence of parameter uncertainty in the MASs, an auxiliary function is designed to compensate the impact of uncertainty. Afterwards, an adaptive controller with dynamic coupling gain is designed based on observation errors and auxiliary function. Subsequently, it is demonstrated that the MASs with parameter uncertainty and leader of nonzero input can achieve bipartite consensus by stability analysis. Finally, the effectiveness of the proposed scheme is verified through simulation.**

***Index Terms*—Multi-agent systems, parameter uncertainty, leader of nonzero input, adaptive control, bipartite consensus**

I. Introduction

In recent years, multi-agent systems (MASs) have received increasing attention due to their widespread applications in the fields of science and engineering, and have become a very important research direction [1–6]. The agent in MASs interacts with others through network communication topology to jointly achieve specified control objectives. Compared to a single system, MASs have stronger robustness and better controllability. The consensus control of MASs is an important research topic. With the development of control theory, many scholars have designed corresponding control protocols for the consensus problem, ultimately achieving the consensus of MASs [7–13].

It is noticed that the aforementioned works in [7–13] are very profound and meaningful, but the relationships between the considered agents are all cooperative. However, there is also a competitive relationship between some agents in some practical systems, such as social networks [14] and unmanned systems [15]. Therefore, it is very crucial to achieve consensus in MASs under cooperative and competitive relationships. The author in Ref. [16] considered the existence of cooperative and competitive relationships among agents and first proposed the concept of bipartite consensus. Afterwards, some scholars conducted research on the basis of Ref. [16] and achieved fruitful research results [17–23]. The author in Ref. [18] utilized the output information of neighbour agents to design an observer, and a dynamic event-triggered controller was designed to achieve the bipartive consensus of discrete-time MASs under competitive networks. In Ref. [20], the bipartive consensus of MASs with leader-followers was studied, and a hybrid-triggered controller combining time-triggered and event-triggered controller was proposed to enable the followers to track the leader's trajectory in bipartive manner. The author in Ref. [22] combined mechanical models with MASs to study the bipartite consensus problem of the system. However, these literatures are focused on leaderless or autonomous leader systems, ignoring the presence of additional inputs from leaders, which have certain limitations.

In addition, since the presence of leader inputs is inevitable in actual systems, and sometimes input signals are unknown and time-varying [24], how to solve this problem is challenging. Some scholars utilized the assumption that unknown input signals are bounded, and subsequently designed control protocols to achieve consensus of MASs [25–28]. In Ref. [24], the consensus tracking problem of first-order nonlinear leader-follower MASs was studied, where the leader has an unknown external input signal that is unknown to all followers. The author in Ref. [27] studied a class of multipe input multiple output (MIMO) linear MASs with leader nonzero input signals under switching topology. The consensus tracking of the system is achieved under the assumption that the leader's nonzero input signal is bounded. Moreover, during the operation of the system, uncertainty may arise due to modeling errors, measurement errors, parameter fluctuations, and other factors. The existence of uncertainty seriously affects the stable operation of the system, and more seriously, it can cause irreversible losses. Therefore, it is very necessary to deal with the parameter uncertainty in the system to ensure the stable operation of the system [29–32]. In Ref. [31], an adaptive fault-tolerant controller was designed to effectively address the impact of mismatched parameter

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uncertainties and actuator faults in leader-follower MASs. For the tracking problem of linear MASs with parameter model uncertainty and time-varying communication delay, the author in Ref. [32] proposed a delayed distributed proportional integral derivative controller to handle the effects of uncertainty and communication delay. Although these literatures address the issue of parameter uncertainty, they only consider the cooperative relationship between agents, and this has certain limitations. Hence, it is necessary to study the bipartite consensus of MASs with parameter uncertainty and non-autonomous leader under cooperative and competitive relationships. This is also the research motivation of this article.

To sum up, this article investigates the bipartite consensus of MASs with parameter uncertainty and leader of nonzero input under a signed digraph. The main contributions are as follows:

(1) For consensus control problems of MASs with parameter uncertainty, most literatures focus on unsigned graphs [29–32]. In comparison, this article further extends it to signed graph networks with positive and negative weights.

(2) Considering the existence of unknown input signals in the leader, the follower agents are unable to obtain accurate information about the leader. Compared to using the bounds of unknown input signals for analysis, this article designs a distributed leader state observer based on parameterization theory, enabling the follower agent to track the leader's signal.

(3) An auxiliary function is designed, and then an adaptive controller is designed based on the observer and auxiliary function to reduce the influence of unknown inputs and parameter uncertainties of the leader. Ultimately, the bipartite consensus of MASs with parameter uncertainty and leader of nonzero input is achieved.

The structure of this article is as follows. Section II provides the problem statement and some preliminaries. The designed adaptive observer and controller are introduced in Section III. Then, Section IV shows the numerical simulation. Finally, the conclusion is provided in Section V.

Notation The maximal eigenvalue of matrix M is defined as $\lambda_{\max}(M)$. $1_N = [1,1,...,1]$, where N represents the number of follower agents. $\|*\|$ represents the 2-norm of $*$. The symbol diag$\{\cdot\}$ indicates a diagonal matrix. $\dot{*}$ represents the derivative of $*$. I represents an appropriate dimensional identity matrix, and I_N and I_n are N-dimensional and n-dimensional identity matrices, respectively. $\otimes$ represents Kronecker product.

II. Preliminary and Problem Statement

A. Graph Theory

$G(U_g, K_g, A_g)$ is a signed digraph with N nodes, where $U_g = \{v_1, v_2, \ldots, v_N\}$ is the node set, $K_g = \left\{(v_i, v_j) | i \neq j, v_i, v_j \in U_g\right\}$ is the edge set, and $A_g = [a_{ij}] \in \mathbb{R}^{N\times N}$ is the adjacency matrix, where $a_{ij} \neq 0$, if $(v_i, v_j) \in U_g$, and $a_{ij} = 0$, otherwise. In addition, $a_{ij} > 0$ and $a_{ij} < 0$ mean cooperative and competitive relationships between agents. $D_g = \text{diag}\{d_i\}$ is a degree matrix with $d_i = \sum_{j=1}^{N} |a_{ij}|$. $L_g = [l_{ij}] \in \mathbb{R}^{N\times N} = D_g - A_g$ is the Laplacian matrix. If there exists a leader agent labeled as 0, the graph G is converted to $\bar{G}(\bar{U}_g, \bar{K}_g)$, where $\bar{U}_g = U_g \bigcup \{v_0\}$ and $\bar{K}_g = K_g \bigcup \{b_{i,0}\}$. $b_{i,0} > 0$ when the agent i is connected to the leader, otherwise $b_{i,0} = 0$. Next, the pinning matrix is defined as $B_g = \text{diag}\{b_{1,0}, b_{2,0}, \ldots, b_{N,0}\} \in \mathbb{R}^{N\times N}$.

Definition 1 If U_g can be divided into U_g^1 and U_g^2, it is structurally balanced, where $U_g^1 \cup U_g^2 = U_g$ and $U_g^1 \cap U_g^2 = \varnothing$. Thus, $a_{ij} \geqslant 0$, $\forall\, v_i, v_j \in U_g^h$ or $v_i, v_j \in U_g^m$, and $a_{ij} \leqslant 0$, $\forall\, v_i \in U_g^h$ and $v_j \in U_g^m$, where $h \neq m$ $(h, m \in \{1,2\})$. Define a symbol matrix $S = \text{diag}\{s_i\}$, where $s_i = 1$, $\forall\, v_i \in U_g^h$, and $s_i = -1$, $\forall\, v_i \in U_g^m$, where $h \neq m$ [33].

B. Problem Statement

Considering the following MASs, the dynamics of the follower $\dot{x}_i(t)$ is

$$\dot{x}_i(t) = (A + \Delta A(t))x_i(t) + Bu_i(t) \tag{1}$$

where $x_i(t) \in \mathbb{R}^n$, $i = 1, 2, \ldots, N$ is the system state. A and B are constant matrices with appropriate dimensions. $u_i(t) \in \mathbb{R}^r$ is control input of the system. Moreover, the pair (A, B) is stabilizable. $\Delta A(t)$ is the parameter uncertainty of the system.

The dynamics of the leader $\dot{x}_0(t)$ is as follows

$$\dot{x}_0(t) = Ax_0(t) + Br_0(t) \tag{2}$$

where $x_0(t) \in \mathbb{R}^n$ is the state of leader and $r_0(t)$ is the unknown input of leader. Note that only partial followers can receive leader's signal. For the convenience of expression, t is omitted in the subsequent symbol representation.

Definition 2 For MASs in Eqs. (1) and (2), if the following condition is satisfied

$$\lim_{t\to\infty} \|x_i - s_i x_0\| = 0, \quad i = 1, 2, \ldots, N \tag{3}$$

the MASs achieve bipartite consensus, where s_i is shown in Definition 1 [34].

Furthermore, to solve the bipartite consensus of the MASs with parameter uncertainty and leader of nonzero input, Assumptions 1–3 and Lemmas 1 and 2 are given.

Assumption 1 The signed digraph G is structurally balanced, and has a directed spanning tree with the leader as its root [33].

Assumption 2 ΔA satisfies the following matching condition

$$\Delta A = BM_i(t) \tag{4}$$

where $M_i(t)$ is an unknown matrix with $\|M_i(t)\| \leqslant \sigma$. σ is a positive constant [30].

Assumption 3 The unknown input r_0 of the leader is parameterized with a set of base functions as follows

$$r_0 = \theta_0^{\mathrm{T}} \phi_0 \tag{5}$$

where ϕ_0 denotes the base function and θ_0 is a constant parameter vector [34].

Lemma 1 For a signed digraph G satisfying Assumption

1, $H = L_g + B_g$, then $H > 0$, where L_g is the Laplacian matrix and B_g is the pinning matrix between the leader and follower, respectively [35].

Lemma 2 $\forall\, x, y \in \mathbb{R}^m$, where x and y are vectors, there exist two matrices of compatible dimensions M and Q, and a positive-definite matrix $\Lambda > 0$ that satisfy the following inequality [36]

$$2x^{\mathrm{T}} MQy \leqslant x^{\mathrm{T}} M\Lambda M^{\mathrm{T}} x + y^{\mathrm{T}} Q^{\mathrm{T}} \Lambda^{-1} Qy \tag{6}$$

III. Main Result

Due to the presence of parameter uncertainty and unknown input signals of the leader, it poses challenges to the research of bipartive consensus problems. Therefore, we need to design a control scheme to solve this problem.

According to MASs in Eqs. (1) and (2), a bipartite consensus error ϱ_i is defined as follows

$$\varrho_i = x_i - s_i x_0 \tag{7}$$

Due to the presence of unknown input in the leader, the followers are unable to accurately obtain the leader's information. Therefore, a leader state observer η_i is designed to enable the followers to obtain the leader's state.

First, a distributed leader's relative output error ξ_i is defined as follows

$$\begin{aligned}\xi_i = &\sum_{j\in N_i} |a_{ij}|(\eta_i - \mathrm{sign}(a_{ij})\eta_j) + b_{i,0}(\eta_i - x_0) = \\ &\sum_{j\in N_i} |a_{ij}|(\mu_i - \mathrm{sign}(a_{ij})\mu_j) + b_{i,0}\mu_i\end{aligned} \tag{8}$$

where $\mu_i = \eta_i - x_0$ is the error between the observer and the leader agent, and N_i is the neighbor of the agent i.

Based on Eq. (8), the leader state observer is designed in the following form

$$\dot{\eta}_i = A\eta_i + B\hat{\theta}_i^{\mathrm{T}}\phi_0 - \rho BB^{\mathrm{T}} P_1 \xi_i \tag{9}$$

$$\dot{\hat{\theta}}_i = \tau_1(-\phi_0 \varepsilon_i^{\mathrm{T}} P_1 B - \kappa_1 \alpha(t)\hat{\theta}_i) \tag{10}$$

where $\hat{\theta}_i$ is the estimation of θ_0 by each follower agent. It has an estimation error $\tilde{\theta}_i = \hat{\theta}_i - \theta_0$. ρ, τ_1, and κ_1 are positive constants. ε_i is the bipartite observer error of follower agents. P_1 is a positive-definite matrix. $\alpha(t)$ is an exponential function with a natural constant e as the base and decreasing over time. It is worth noting that $\int_{t_0}^{t}\alpha(t)\mathrm{d}\tau \leqslant \delta$, the initial moment t_0 satisfying $t_0 \geqslant 0$, where constant $\delta > 0$. Moreover, if $\xi_i = 0$, then $\mu_i = 0$ can be obtained, which means that the observer can accurately observe the leader's state.

Then, the bipartite observer error of follower agents ε_i is defined as follows

$$\varepsilon_i = x_i - s_i \eta_i \tag{11}$$

Note that if $\lim_{t\to\infty}\xi_i = 0$ and $\lim_{t\to\infty}\varepsilon_i = 0$, it can be obtained that $\lim_{t\to\infty}\varrho_i = 0$, which means that the MASs achieve bipartive consensus.

Next, the dynamics equations of the error system are described as Eqs. (12) and (13)

$$\dot{\mu}_i = \dot{\eta}_i - \dot{x}_0 = A\mu_i + B\tilde{\theta}_i^{\mathrm{T}}\phi_0 - \rho BB^{\mathrm{T}} P_1 \xi_i \tag{12}$$

$$\begin{aligned}\dot{\varepsilon}_i = &\dot{x}_i - s_i\dot{\eta}_i = \\ &A\varepsilon_i + BM_i(t)x_i + Bu_i - s_i B\hat{\theta}_i^{\mathrm{T}}\phi_0 + \\ &s_i\rho BB^{\mathrm{T}} P_1 \xi_i\end{aligned} \tag{13}$$

Let $x = \mathrm{col}(x_i)$, $\mu = \mathrm{col}(\mu_i)$, $\xi = \mathrm{col}(\xi_i)$, $\varepsilon = \mathrm{col}(\varepsilon_i)$, $u = \mathrm{col}(u_i)$, $\tilde{\Theta} = \mathrm{diag}\{\tilde{\theta}_i\}$, $\hat{\Theta} = \mathrm{diag}\{\hat{\theta}_i\}$, $\Phi = 1_N\phi_0$, and $M = \mathrm{diag}\{M_i(t)\}$, then we get

$$\begin{aligned}\dot{\xi} = &(H\otimes I_n)\dot{\mu} = \\ &(I_N\otimes A)\xi + (H\otimes B)\tilde{\Theta}^{\mathrm{T}}\Phi - \\ &\rho(H\otimes BB^{\mathrm{T}} P_1)\xi\end{aligned} \tag{14}$$

$$\begin{aligned}\dot{\varepsilon} = &(I_N\otimes A)\varepsilon + (I_N\otimes B)u - (S\otimes B)\hat{\Theta}_i^{\mathrm{T}}\Phi + \\ &(S\otimes B)Mx + \rho(S\otimes BB^{\mathrm{T}} P_1)\xi\end{aligned} \tag{15}$$

Then, the adaptive controller with a dynamic coupling gain is designed as

$$u_i = -c_i B^{\mathrm{T}} P_2 \varepsilon_i + s_i\hat{\theta}_i^{\mathrm{T}}\phi_0 + \Gamma_i \tag{16}$$

$$\dot{c}_i = \tau_3(\varepsilon_i^{\mathrm{T}} P_2 BB^{\mathrm{T}} P_2 \varepsilon_i) \tag{17}$$

where c_i is a dynamic coupling gain and $c_i(0) > 0$. τ_3 is a positive constant. P_2 is a positive-defined matrix. Γ_i is an auxiliary function to compensate the impact of parameter uncertainty in Eq. (1), and it has the following expression

$$\Gamma_i = -\frac{\hat{q}_i^2(B^{\mathrm{T}} P_2\varepsilon_i)\|x_i\|^2}{\hat{q}_i\|\varepsilon_i^{\mathrm{T}} P_2 B\|\|x_i\| + \alpha(t)} \tag{18}$$

$$\dot{\hat{q}}_i = \tau_2(\|\varepsilon_i^{\mathrm{T}} P_2 B\|\|x_i\| - \kappa_2\alpha(t)\hat{q}_i) \tag{19}$$

where τ_2 and κ_2 are positive constants. $\hat{q}_i$ is the estimation of σ. Define a vector $\tilde{q}_i = \hat{q}_i - \sigma$.

Next, the main result of this article will be presented.

Theorem 1 Under Assumptions 2 and 3, consider Eqs. (1) and (2), the observer in Eqs. (9) and (10), and the adaptive controller in Eqs. (16)–(19). If there exist positive-definite matrices P_1 and P_2 and positive constants β_1, β_2, and c_0, which satisfy the following conditions

$$AP_1^{\mathrm{T}} + P_1 A - P_1^{\mathrm{T}} BB^{\mathrm{T}} P_1 + \beta_1 I = 0 \tag{20}$$

$$AP_2^{\mathrm{T}} + P_2 A - (2c_0 - \rho)P_2^{\mathrm{T}} BB^{\mathrm{T}} P_2 + \beta_2 I = 0 \tag{21}$$

the bipartive consensus of MASs with parameter uncertainty and leader of nonzero input is arrived.

Proof The following Lyapunov function V is constructed

$$\begin{aligned}V = &\xi^{\mathrm{T}}(H^{-1}\otimes P_1)\xi + \varepsilon^{\mathrm{T}}(I_N\otimes P_2)\varepsilon + \\ &\sum_{i=1}^{N}\frac{1}{\tau_1}\tilde{\theta}_i^{\mathrm{T}}\tilde{\theta}_i + \sum_{i=1}^{N}\frac{1}{\tau_2}\tilde{q}_i^{\mathrm{T}}\tilde{q}_i + \sum_{i=1}^{N}\frac{1}{\tau_3}(c_i - c_0)^2\end{aligned} \tag{22}$$

Then, the time derivation of V along Eqs. (14) and (15) is as follows

$$\begin{aligned}\dot{V} &= \dot{\xi}^{\mathrm{T}}(H^{-1}\otimes P_1)\xi + \xi^{\mathrm{T}}(H^{-1}\otimes P_1)\dot{\xi} + \dot{\varepsilon}^{\mathrm{T}}(I_N\otimes P_2)\varepsilon + \\ &\varepsilon^{\mathrm{T}}(I_N\otimes P_2)\dot{\varepsilon} + \sum_{i=1}^{N}\frac{2}{\tau_1}\tilde{\theta}_i^{\mathrm{T}}\dot{\hat{\theta}}_i + \sum_{i=1}^{N}\frac{2}{\tau_2}\tilde{q}_i^{\mathrm{T}}\dot{\hat{q}}_i + \\ &\sum_{i=1}^{N}\frac{2}{\tau_3}(c_i-c_0)\dot{c}_i = \xi^{\mathrm{T}}(H^{-1}\otimes(A^{\mathrm{T}}P_1+P_1A))\xi + \\ &2\xi(I_N\otimes P_1B)\tilde{\Theta}^{\mathrm{T}}\Phi - 2\rho\xi^{\mathrm{T}}(I_N\otimes P_1BB^{\mathrm{T}}P_1)\xi + \\ &\varepsilon^{\mathrm{T}}(I_N\otimes(A^{\mathrm{T}}P_2+P_2A))\varepsilon + 2\varepsilon^{\mathrm{T}}(I_N\otimes P_2B)u - \\ &2\varepsilon^{\mathrm{T}}(S\otimes P_2B)\hat{\Theta}^{\mathrm{T}}\Phi + 2\rho\varepsilon^{\mathrm{T}}(S\otimes P_2BB^{\mathrm{T}}P_1)\xi + \\ &2\varepsilon^{\mathrm{T}}(I_N\otimes P_2B)Mx + \sum_{i=1}^{N}\frac{2}{\tau_1}\tilde{\theta}_i^{\mathrm{T}}\dot{\hat{\theta}}_i + \\ &\sum_{i=1}^{N}\frac{2}{\tau_2}\tilde{q}_i^{\mathrm{T}}\dot{\hat{q}}_i + \sum_{i=1}^{N}\frac{2}{\tau_3}(c_i-c_0)\dot{c}_i\end{aligned} \tag{23}$$

Based on the expression of the controller in Eq. (16) and auxiliary function in Eq. (18), then Eq. (23) can be converted to the following expression

$$\begin{aligned}\dot{V} &\leqslant \xi^{\mathrm{T}}(H^{-1}\otimes(A^{\mathrm{T}}P_1+P_1A))\xi - \sum_{i=1}^{N}(2\rho\xi_i^{\mathrm{T}}P_1BB^{\mathrm{T}}P_1\xi_i + \\ &\varepsilon_i^{\mathrm{T}}(A^{\mathrm{T}}P_2+P_2A)\varepsilon_i - 2c_i\varepsilon_i^{\mathrm{T}}P_2BB^{\mathrm{T}}P_2\varepsilon_i - \\ &2s_i\rho\varepsilon_i^{\mathrm{T}}P_2BB^{\mathrm{T}}P_1\xi + 2\varepsilon_i^{\mathrm{T}}P_2BM_i(t)x_i + \\ &\frac{2\hat{q}_i^2\|\varepsilon_i^{\mathrm{T}}P_2B\|^2\|x_i\|^2}{\hat{q}_i\|\varepsilon_i^{\mathrm{T}}P_2B\|\|x_i\|+\alpha(t)}) + \sum_{i=1}^{N}(\frac{2}{\tau_2}\tilde{q}_i^{\mathrm{T}}\dot{\hat{q}}_i - \\ &2\kappa_1\alpha(t)\tilde{\theta}_i^{\mathrm{T}}\hat{\theta}_i + \frac{2}{\tau_3}(c_i-c_0)\dot{c}_i)\end{aligned} \tag{24}$$

Since we have

$$2\varepsilon_i^{\mathrm{T}}P_2BM_i(t)x_i \leqslant 2\sigma\|\varepsilon_i^{\mathrm{T}}P_2B\|\|x_i\| \tag{25}$$

based on Lemma 2 and the definition of s_i, we can obtain Formula (26)

$$\begin{aligned}&2s_i\rho\varepsilon_i^{\mathrm{T}}P_2BB^{\mathrm{T}}P_1\xi_i \leqslant \\ &\quad\rho(\varepsilon_i^{\mathrm{T}}P_2BB^{\mathrm{T}}P_2\varepsilon + \xi_i^{\mathrm{T}}P_1Bs_i^2B^{\mathrm{T}}P_1\xi_i) \leqslant \\ &\quad\rho(\varepsilon_i^{\mathrm{T}}P_2BB^{\mathrm{T}}P_2\varepsilon + \xi_i^{\mathrm{T}}P_1BB^{\mathrm{T}}P_1\xi_i)\end{aligned} \tag{26}$$

After that, let $\rho \geqslant \lambda_{\max}(H^{-1})$, and based on adaptive law in Eq. (17), one obtains

$$\begin{aligned}\dot{V} &\leqslant \xi^{\mathrm{T}}(H^{-1}\otimes(A^{\mathrm{T}}P_1+P_1A-P_1BB^{\mathrm{T}}P_1))\xi + \\ &\sum_{i=1}^{N}(\varepsilon_i^{\mathrm{T}}(A^{\mathrm{T}}P_2+P_2A-(2c_0-\rho)P_2BB^{\mathrm{T}}P_2)\varepsilon_i + \\ &2\sigma\|\varepsilon_i^{\mathrm{T}}P_2B\|\|x_i\| - \frac{2\hat{q}_i^2\|\varepsilon_i^{\mathrm{T}}P_2B\|^2\|x_i\|^2}{\hat{q}_i\|\varepsilon_i^{\mathrm{T}}P_2B\|\|x_i\|+\alpha(t)}) + \\ &\sum_{i=1}^{N}(\frac{2}{\tau_2}\tilde{q}_i^{\mathrm{T}}\dot{\hat{q}}_i - 2\kappa_1\alpha(t)\tilde{\theta}_i^{\mathrm{T}}\hat{\theta}_i)\end{aligned} \tag{27}$$

Then, according to Eq. (19), one further gets

$$\begin{aligned}\dot{V} &\leqslant \xi^{\mathrm{T}}(H^{-1}\otimes(A^{\mathrm{T}}P_1+P_1A-P_1BB^{\mathrm{T}}P_1))\xi + \\ &\sum_{i=1}^{N}\Big(\varepsilon_i^{\mathrm{T}}(A^{\mathrm{T}}P_2+P_2A-(2c_0-\rho)P_2BB^{\mathrm{T}}P_2)\varepsilon_i + \\ &2\hat{q}_i\|\varepsilon_i^{\mathrm{T}}P_2B\|\|x_i\| - \frac{2\hat{q}_i^2\|\varepsilon_i^{\mathrm{T}}P_2B\|^2\|x_i\|^2}{\hat{q}_i\|\varepsilon_i^{\mathrm{T}}P_2B\|\|x_i\|+\alpha(t)}\Big) - \\ &\sum_{i=1}^{N}(2\kappa_2\alpha(t)\tilde{q}_i^{\mathrm{T}}\hat{q}_i + 2\kappa_1\alpha(t)\tilde{\theta}_i^{\mathrm{T}}\hat{\theta}_i)\end{aligned} \tag{28}$$

Note that

$$\begin{aligned}&2\hat{q}_i\|\varepsilon_i^{\mathrm{T}}P_2B\|\|x_i\| - \frac{2\hat{q}_i^2\|\varepsilon_i^{\mathrm{T}}P_2B\|^2\|x_i\|^2}{\hat{q}_i\|\varepsilon_i^{\mathrm{T}}P_2B\|\|x_i\|+\alpha(t)} = \\ &\qquad\frac{2\hat{q}_i^2\|\varepsilon_i^{\mathrm{T}}P_2B\|\|x_i\|\alpha(t)}{\hat{q}_i\|\varepsilon_i^{\mathrm{T}}P_2B\|\|x_i\|+\alpha(t)} \leqslant 2\alpha(t)\end{aligned} \tag{29}$$

Consequently, we can obtain

$$\begin{aligned}\dot{V} &\leqslant \xi^{\mathrm{T}}(H^{-1}\otimes(A^{\mathrm{T}}P_1+P_1A-P_1BB^{\mathrm{T}}P_1+\beta_1I))\xi + \\ &\sum_{i=1}^{N}\varepsilon_i^{\mathrm{T}}(A^{\mathrm{T}}P_2+P_2A-(2c_0-\rho)P_2BB^{\mathrm{T}}P_2+\beta_2I)\varepsilon_i - \\ &\sum_{i=1}^{N}(2\kappa_2\alpha(t)\tilde{q}_i^{\mathrm{T}}\hat{q}_i + 2\kappa_1\alpha(t)\tilde{\theta}_i^{\mathrm{T}}\hat{\theta}_i + 2\alpha(t)) - \\ &\beta_1\xi_i^{\mathrm{T}}(H^{-1}\otimes I_n)\xi - \beta_2\varepsilon^{\mathrm{T}}\varepsilon\end{aligned} \tag{30}$$

Since we have

$$-\tilde{\theta}_i^{\mathrm{T}}\hat{\theta}_i = -\tilde{\theta}_i^{\mathrm{T}}(\tilde{\theta}_i+\theta_0) \leqslant \frac{1}{4}\theta_0^2 \tag{31}$$

$$-\tilde{q}_i\hat{q}_i = -\tilde{q}_i(\tilde{q}_i+\sigma) \leqslant \frac{1}{4}\sigma^2 \tag{32}$$

based on Eqs. (20) and (21), and combining Eqs. (31) and (32) to Formula (30), we can obtain

$$\dot{V} \leqslant -\beta_1\xi^{\mathrm{T}}(H^{-1}\otimes I_n)\xi - \beta_2\varepsilon^{\mathrm{T}}\varepsilon + \sum_{i=1}^{N}\hat{\kappa}\alpha(t) \tag{33}$$

where $\hat{\kappa} = 2 + \frac{\kappa_1}{2}\theta_0^2 + \frac{\kappa_2}{2}\sigma^2$.

Then, the following inequality can be obtained

$$\begin{aligned}V(t) &\leqslant V(t_0) - \int_{t_0}^{t}\beta_1\xi^{\mathrm{T}}(H^{-1}\otimes I_n)\xi\mathrm{d}\tau - \\ &\int_{t_0}^{t}\beta_2\varepsilon^{\mathrm{T}}\varepsilon\mathrm{d}\tau + \int_{t_0}^{t}\sum_{i=1}^{N}\hat{\kappa}\alpha(t)\mathrm{d}\tau \leqslant \\ &V(t_0) + \sum_{i=1}^{N}\hat{\kappa}\delta\end{aligned} \tag{34}$$

which means that $V(t)$ is uniformly bounded, so ξ_i and ε_i are also uniformly bounded.

Furthermore, based on the fact that $V(t) \geqslant 0$, it follows from Formula (34) that

$$\int_{t_0}^{t}\beta_1\xi^{\mathrm{T}}(H^{-1}\otimes I_n)\xi\mathrm{d}\tau \leqslant V(t_0) + \sum_{i=1}^{N}\hat{\kappa}\delta \tag{35}$$

$$\int_{t_0}^{t}\beta_2\varepsilon^{\mathrm{T}}\varepsilon\mathrm{d}\tau \leqslant V(t_0) + \sum_{i=1}^{N}\hat{\kappa}\delta \tag{36}$$

Applying the Barbalat lemma to Formulas (35) and (36), it can yield $\lim_{t\to\infty}\|\xi_i\|=0$ and $\lim_{t\to\infty}\|\varepsilon_i\|=0$. According to Eqs. (7) and (11), we get $\lim_{t\to\infty}\|\varrho_i\|=0$. This means that MASs with parameter uncertainty and the leader unknown input achieve the bipartite consensus.

The proof is completed. ■

Remark 1 The existence of uncertainty seriously affects the stable operation of the system, and more importantly, it can cause irreversible losses. Therefore, this article studies the MASs with parameter uncertainty. Not only that, but also the cleaning situation with nonzero input of the leader is tested. The author in Refs. [24, 30] only considered the cooperative relationship between agents, but ignored the competitive relationship. Therefore, this study further extends the traditional consistency problem to the two-part consistency problem in signed directed graphs with different weight information. This has more practical significance.

Remark 2 In practical systems, it is inevitable for leader to have external input. However, current literatures need to use the assumption of the upper bound of unknown input [24–28] and are conducted in an unsigned graph without considering the competitive relationship between agents, which have significant limitations. Therefore, this article designs an adaptive learning rate to estimate the unknown input signal of the leader by parameterization theory. This does not require knowing the upper bound of the unknown input. Afterwards, a leader state observer is designed to enable followers to accurately obtain the leader's signal.

Remark 3 The parameter uncertainty in the system can affect its performance. This article utilizes bipartite observer error signals and the state information of the agent itself, and designs an auxiliary function to compensate the impact of parameter uncertainty by introducing an exponential function. Furthermore, an adaptive controller is designed to enable the MASs to achieve bipartite consensus with parameter uncertainty and nonzero input of leader under a signed digraph. In addition, the control scheme designed in this article has a wide range of applications. It is also applicable to the case that there is no parameter uncertainty in the system.

IV. Numerical Simulation

This section will present a simulation example to verify the effectiveness of the proposed solution. Based on Eqs. (1) and (2), the parameters for simulation are given as follows

$$A=\begin{bmatrix}0 & 2\\ -2 & 0\end{bmatrix},\ B=\begin{bmatrix}1\\ 1\end{bmatrix},\ \Delta A=\begin{bmatrix}0 & \sin(t)\\ 0 & \sin(t)\end{bmatrix}.$$

The adjacency matrix A_g of the structurally balanced signed digraph G is

$$A_g=\begin{bmatrix}0 & 2 & 0 & 0\\ 0 & 0 & -1 & 0\\ 0 & 0 & 0 & 2\\ -1 & 0 & 3 & 0\end{bmatrix}.$$

The node sets are divided into $V_1=\{1,2\}$ and $V_2=\{3,4\}$. The unknown input signal of leader is $r_0=0.5\sin(t)$. The initial state vector of leader agent is $x_0(0)=[0.20\ \ -0.10]^{\mathrm{T}}$. The initial values of follower agents are $x_1(0)=[1.30\ \ 0.20]^{\mathrm{T}}$, $x_2(0)=[-1.20\ \ 1.10]^{\mathrm{T}}$, $x_3(0)=[0.80\ \ 0.55]^{\mathrm{T}}$, and $x_4(0)=[1.30\ \ 0.30]^{\mathrm{T}}$. The initial values of observer are selected as $\eta_1(0)=[0.70\ \ -0.20]^{\mathrm{T}}$, $\eta_2(0)=[-1.60\ \ 0.20]^{\mathrm{T}}$, $\eta_3(0)=[0.20\ \ 0.75]^{\mathrm{T}}$, and $\eta_4(0)=[1.80\ \ -1.30]^{\mathrm{T}}$. Moreover, the other parameters are as follows $c_0=4$, $\rho=3$, $\beta_1=4.52$, $\beta_2=2.25$, $\kappa_1=20.0$, $\kappa_2=1.2$, $\kappa_3=0.5$, $\tau_1=200.0$, $\tau_2=1.5$, $\tau_3=0.5$, and $\alpha(t)=\mathrm{e}^{-2t}$. The initial values of the adaptive laws are selected as $c_1(0)=1.2$, $c_2(0)=0.8$, $c_3(0)=1.0$, $c_4(0)=1.5$, $q_1(0)=1.0$, $q_2(0)=0.4$, $q_3(0)=1.5$, and $q_4(0)=0.8$.

By solving Eqs. (20) and (21), we can obtain

$$P_1=\begin{bmatrix}3.5655 & -0.7925\\ -0.7925 & 1.9544\end{bmatrix},$$

$$P_2=\begin{bmatrix}1.3733 & -0.4662\\ -0.4662 & 0.7438\end{bmatrix}.$$

Next, the simulation results are provided in Figs. 1–6. Figure 1 shows the trajectories of leader and follower agents. The trajectories of leader and observers are drawn in Fig. 2. It can be seen that the followers can track the trajectory of leader. Figures 3 and 4 show the trajectories of the error signals μ_i and ε_i, respectively. The trajectories of coupling gain c_i and $\hat{q}_i$ are shown in Figs. 5 and 6, respectively. Hence, the bipartite consensus of MASs with parameter uncertainty and leader of nonzero input is achieved.

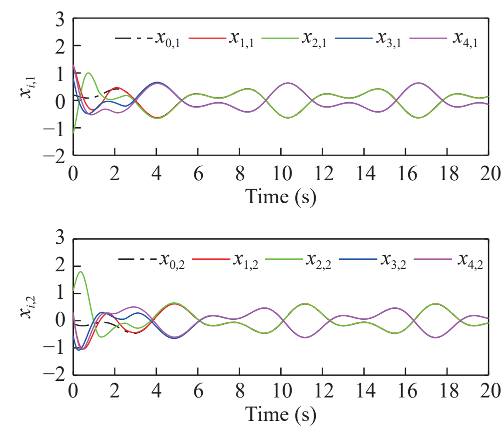

Figure 1 Trajectories of the leader x_0 and followers x_i.

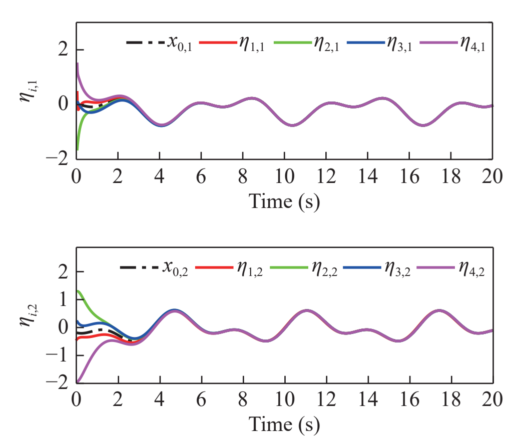

Figure 2 Trajectories of the leader x_0 and observers η_i.

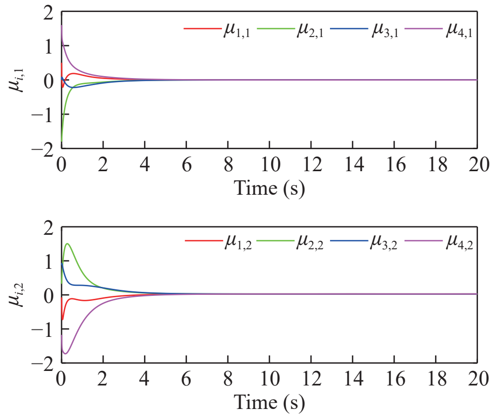

Figure 3 Trajectories of the error μ_i.

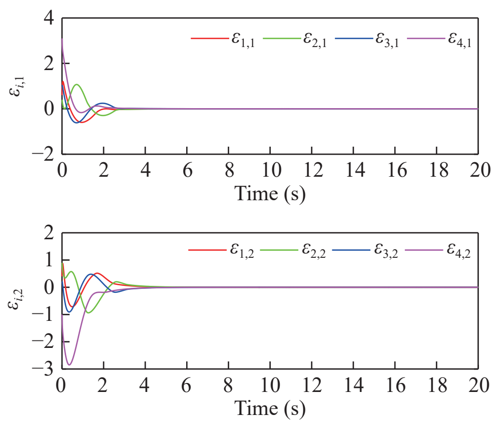

Figure 4 Trajectories of the error ε_i.

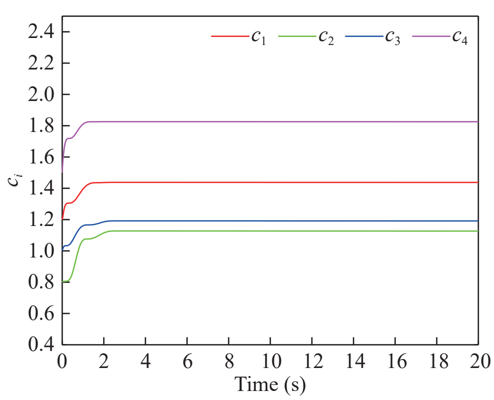

Figure 5 Trajectories of the coupling gain c_i.

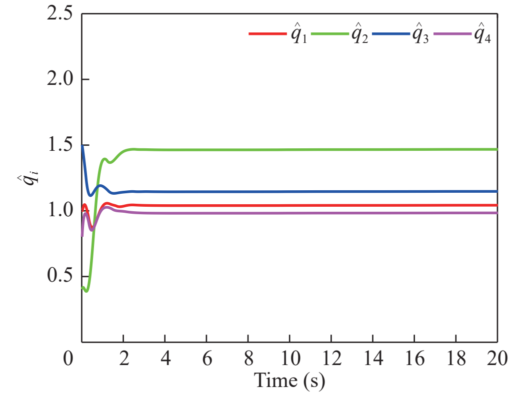

Figure 6 Trajectories of $\hat{q}_i$.

V. CONCLUSION

This article studies the bipartite consensus of MASs with parameter uncertainty and leader of nonzero input on a signed digraph. We design a leader state observer to overcome the issue of information exchange between agents caused by unknown input signal of the leader. Afterwards, an auxiliary function is designed to deal with the impact of parameter uncertainty. Then, an adaptive controller with dynamic coupling gain based on bipartite observer errors and auxiliary function is further designed to achieve the bipartite consensus of MASs. Finally, we verify the effectiveness of the proposed scheme by simulation. The control scheme designed in this article is aimed at homogeneous systems. In the future, we will continue to study the finite-time consensus of heterogeneous MASs.

ACKNOWLEDGMENT

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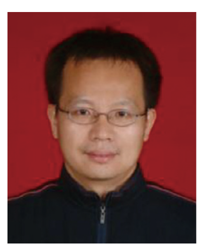

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