Cluster Synchronization of Fractional-Order Complex Community Networks in Quaternion-Valued Field

Bibo Zheng and Zhanshan Wang

Abstract—**In this paper, cluster synchronization of fractional-order complex community networks (FOCCNs) in quaternion-valued filed is addressed based on the non-separation approach. To carry out tasks with different requirements, quaternion-valued FOCCNs are divided into several clusters, each of which owns different dynamic behaviors. Considering the interaction of nodes in the cluster and the influence of external driving, a hybrid controller is designed, in which the coupling strength and feedback control gains are simultaneously regulated with the evolution of states. To establish the cluster synchronization criteria for the quaternion-valued FOCCNs, some analytical approaches, such as the mean value theorem and the proof by contradiction are utilized. Furthermore, for the asymptotical synchronization of fractional-order complex networks, the case with identical nodes is also considered as a special case. Finally, two quaternion-valued FOCCNs are divided to synchronize to the desired trajectory, and a numerical simulation shows the validity of the proposed approach.**

Index Terms—**Fractional-order, community complex network, synchronization, adaptive strategy, quaternion-valued**

I. Introduction

The 21st century is represented by complexity, and the complex system has gradually developed into a hot interdisciplinary topic with the vigorous development of scientific research. Complex network is made up of numerous interconnected nodes and edges, and it is a network model connected by different dynamic nodes based on a certain topological relationship. In the real world, the power grid system, biological network, and social network can be characterized by complex network [1–3]. Currently, the theoretical study of complex network has raised a research fever in the fields of mathematics, physics, computer science, neuroscience, and other research fields.

With the advancement of fractional calculus, it shows many advantages over integral calculus. Because fractional differential equation has the characteristics of heredity, memory, and more degrees of freedom, its mathematical model is more complete and adequate [4]. Currently, fractional-order differential equation has been used for modeling in viscoelastic systems, biomedical and material sciences, and so on [5–7]. Considering all the aspects mentioned above, numerous studies have attempted to incorporate fractional calculus into complex networks, i.e., fractional-order complex networks (FOCNs) [8]. As an interesting collective behavior, synchronization has a lot of applications in many fields, such as secure communication, neuroscience, and biological ecosystems [9, 10]. Nowadays, many outstanding results about synchronization of FOCNs have been reported, such as adaptive synchronization [11], pinning synchronization [12, 13], quasi-synchronization [14], and finite-time synchronization [15, 16].

It is worth noticing that topological structures of all nodes are identical in papers mentioned above. In practice, since the distribution of edges in the network is not always uniform, it is theoretically inevitable that nodes will exhibit different characteristics, which implies the existence of network community structure. A community is defined as a sub-collection of nodes of FOCNs, and the nodes within the same community share a coincident dynamic attribute. In contrast, nodes belonging to different communities exhibit significant differences in their dynamics [17, 18]. Currently, a number of phenomena have indicated the universality of community network mechanisms, such as cellular metabolic network, protein interaction network, and gene regulatory network. Hence, cluster synchronization is recognized as a critical part of synchronization problems of fractional-order complex community networks (FOCCNs). To date, several studies about cluster synchronization of FOCNs have been published [16, 19–22]. By updating feedback control gains, some cluster synchronization criteria for FOCNs with different nodes have been derived in Refs. [20, 21]. In Ref. [16], by designing effective control strategies, finite-time cluster synchronization of FOCNs with nonlinear coupling has been addressed. In Ref. [22], asymptotic and finite-time cluster synchronization for FOCCNs has been investigated by applying external controller, and three community structures have been divided.

However, previous research has mainly focused on the case that parameters of the network are all constants, often overlooking the changes presented in different environments. In reality, the suitable adaptive laws for time-varying coupling strength can be considered, so that the system has the ability to resist to this change [23, 24]. It is worth emphasizing, that although adaptive coupling may ensure the synchronization of the network, orbits of the nodes are sensitive to initial values. Furthermore, adaptive strategies for coupling strength are implemented by acquiring the local dynamic information of nodes, which has little effect on the whole network dynamics. Hence, with the help of external information, feedback control

Manuscript received: 18 October 2023; revised: 26 February 2024; accepted: 3 May 2024. *(Corresponding author: Zhanshan Wang.)*

Citation: B. Zheng and Z. Wang, Cluster synchronization of fractional-order complex community networks in quaternion-valued field, *IJICS*, 2024, 29(2), 71–78.

Bibo Zheng and Zhanshan Wang are with College of Information Science and Engineering, Northeastern University, Shenyang 110819, China (e-mail: zbbenergy@163.com; zhanshan_wang@163.com).

Digital Object Identifier 10.62678/IJICS202403.10109

also can be applied to FOCCNs to regulate the dynamics of the whole network.

Note that real-valued or complex-valued FOCCNs have found successful applications across various domains, including associative memory, parallel computing, and quantum communication. However, they encounter certain limitations when addressing three-dimensional affine transformations [25]. It has been revealed that quaternion offers a direct method for encoding and processing high-dimensional data, significantly enhancing processing efficiency. This is particularly evident in the realm of three-dimensional space transformations. Recently, many domains such as color image compression, computer graphics, color low-light night vision, and robot motion control can be expressed by quaternion-valued networks [25–27]. Hence, it is an interesting problem to investigate synchronization of quaternion-valued FOCCNs.

Inspired by the aforementioned analysis, cluster synchronization is investigated for quaternion-valued FOCCNs via hybrid control approach. In other words, the coupling strength and feedback control gains are adjusted together to coordinate the whole networks to realize synchronization. The contributions in this paper are summarized as follows:

(1) The cluster synchronization of quaternion-valued FOCCNs is directly analyzed without splitting the networks into four real-valued systems, thereby reducing the complexity of analysis.

(2) Considering the interaction of nodes in the cluster and the influence of external control on the quaternion-valued FOCCNs, the hybrid adaptive strategies consisting of coupling strength and feedback control gains are simultaneously applied to the FOCCNs.

Notation Let $\mathbb{R}$ and $\mathbb{R}^n$ be the set of real numbers and the space of real n-dimensional vectors. Q and H^n represent the set of quaternion numbers and a space of real n-dimensional quaternion-valued vectors. $R^{n\times n}$ and $H^{n\times n}$ are the $n\times n$ real-valued matrix and quaternion-valued matrix. I_n represents the n-dimensional identity matrix. For matrix Q, $\bar{Q}$ and Q^{T} denote the conjugate and transpose. Q^* denotes the conjugate transpose. $\lambda_{\mathrm{M}}(Q)$ and $\lambda_{\mathrm{m}}(Q)$ represent the largest and the smallest eigenvalues. For a quaternion number z, $z=z^{\mathrm{R}}+z^{\mathrm{I}}\imath+z^{\mathrm{J}}\jmath+z^{\mathrm{K}}k$, in which z^{R}, z^{I}, z^{J}, and $z^{\mathrm{K}}\in\mathbb{R}$, imaginary units $\imath$, $\jmath$, and k obey Hamilton rules: $\imath\jmath=k=-\jmath\imath$, $\jmath k=\imath=-k\jmath$, $k\imath=\jmath=-\imath k$, and $\imath^2=\jmath^2=k^2=-1$. $\bar{z}=z^{\mathrm{R}}-z^{\mathrm{I}}\imath-z^{\mathrm{J}}\jmath-z^{\mathrm{K}}k$ is the conjugate of z. The undirected graph is denoted by $\mathcal{G}=(\mathcal{V},\mathcal{E})$, in which the vertex set numbered by $\{1,2,\ldots,N\}$ is represented by $\mathcal{V}$, and the undirected edges are presented by $\mathcal{E}=\{(i,j),\ i,j\in\mathcal{V}\}$. If there exists an edge between nodes i and j, then coupling matrix $H_{ij}=H_{ij}\ (i\neq j)$; otherwise, $H_{ij}=H_{ji}=0$ and $H_{ii}=-\sum\limits_{i=1,i\neq j}^{N}H_{ij}$. $\mathcal{N}=\{1,2,\ldots,N\}$.

II. Preliminary and Model Description

A. Problem Formulation

The model of quaternion-valued FOCNs involving N nodes is depicted as

$$
{}_{0}^{C}D_{t}^{\beta}z_i(t)=f(z_i(t))+\sigma(t)\sum_{j=1}^{N}H_{ij}\Gamma z_j(t)+u_i(t),\quad i\in\mathcal{N} \tag{1}
$$

where $\beta\in(0,1)$, the time-varying coupling strength is represented by $\sigma(t)$, $z_i(t)=(z_{i1}(t),z_{i2}(t),\ldots,z_{in}(t))^{\mathrm{T}}\in H^n$ denotes the state of the i-th node, $f(z_i(t))\in H^n$ is a nonlinear vector function, $u_i(t)$ is the controller to be designed, $H_{ij}\in\mathbb{R}^{N\times N}$ is the coupling matrix, and Γ denotes an inner coupling matrix, respectively.

Next, some useful definitions and lemmas about cluster synchronization are introduced.

Definition 1 [18] For an irreducible matrix $H=(h_{ij})\in\mathbb{R}^{N\times N}\in\mathcal{H}_1$, if $h_{ij}=h_{ji}\geqslant 0\ (i\neq j)$, $h_{ii}=-\sum\limits_{j=1,j\neq i}^{N}h_{ij}$, $i\in\mathcal{N}$.

Definition 2 [18] Matrix $H=(h_{ij})\in\mathbb{R}^{N_1\times N_2}\in\mathcal{H}_2$, if $\sum\limits_{j=1}^{N_2}h_{ij}=0$, $i=1,2,\ldots,N_1$.

To study cluster synchronization, assume that the set of nodes of FOCNs in Eq. (1) can be divided into m clusters, i.e., $\{1,2,\ldots,N\}=C_1\bigcup C_2\bigcup\cdots\bigcup C_k\bigcup\cdots\bigcup C_m$, where $C_1=\{1,2,\ldots,\ell_1\}$, $C_2=\{\ell_1+1,\ell_1+2,\ldots,\ell_2\}$, $C_k=\{\ell_{k-1}+1,\ell_{k-1}+2,\ldots,\ell_k\}$, $C_m=\{\ell_{m-1}+1,\ell_{m-1}+2,\ldots,\ell_m\}$. $\ell_0=0$ and $\ell_m=N$, which satisfies that $\sum\limits_{k=1}^{m}\ell_k=N$, $1\leqslant\ell_k<N$. Let $\mathcal{M}=\{1,2,\ldots,m\}$.

Definition 3 [18] If matrix $H=(h_{ij})\in\mathbb{R}^{N\times N}$ is symmetric,

$$
H=\begin{bmatrix} H_{11} & H_{12} & \cdots & H_{1m}\\ H_{21} & H_{22} & \cdots & H_{2m}\\ \vdots & \vdots & \ddots & \vdots\\ H_{m1} & H_{m2} & \cdots & H_{mm}\end{bmatrix},
$$

where $H=(H_{ij})\in\mathbb{R}^{(\ell_i-\ell_{i-1})\times(\ell_j-\ell_{j-1})}$, $H_{ii}\in\mathcal{H}_1$, and $H_{ij}\in\mathcal{H}_2$, $i,j\in\{1,2,\ldots,m\}$, then $H\in\mathcal{H}_3$.

Considering the relationship between nodes and clusters, the quaternion-valued controlled network in Eq. (1) can be rewritten as

$$
{}_{0}^{C}D_{t}^{\beta}z_i(t)=f_k(z_i(t))+\varDelta\sum_{r=1}^{m}\sum_{j\in C_r}H_{ij}\Gamma z_j(t)+u_i(t),\quad i\in C_k,\ k\in\mathcal{M} \tag{2}
$$

where the coupling matrix $H_{ij}\in\mathcal{H}_3$,

$$
\varDelta=\begin{cases}\sigma(t), & \text{if } i,j\in C_k;\\ 1, & \text{if } i\in C_k,\ j\in C_r,\ k\neq r;\end{cases}
$$

and the evolution of nodes in the k-th community is represented by $f_k(z_i(t))$.

Assumption 1 For any vectors $z_1,z_2\in H^n$, there exists constant $L>0$ such that

$$
(z_1-z_2)^*(f(z_1)-f(z_2))+(f(z_1)-f(z_2))^*(z_1-z_2)\leqslant L(z_1-z_2)^*(z_1-z_2).
$$

Then, assume that the quaternion-valued FOCCNs in Eq. (2) synchronize to the desired trajectories explained as $z_{\ell_0}(t),z_{\ell_0+1}(t),\ldots,z_{\ell_1}(t)\to s_1(t)$ and $z_{\ell_{m-1}+1}(t),z_{\ell_{m-1}+2}(t),\ldots,z_{\ell_m}(t)\to s_m(t)$, which means that $S=(s_1(t),s_2(t),\ldots,s_m(t))\in$

H^n is the desired cluster synchronization state of Eq. (2). Here, $s_k(t)$ can be equilibrium point, periodic orbit, or chaotic attractor, which can be depicted by

$$ {}_0^C D_t^\beta s_k(t) = f(s_k(t)), \quad k \in \mathcal{M} \tag{3} $$

Definition 4 [22] The quaternion-valued FOCCNs in Eq. (2) are said to realize cluster synchronization if the desired orbit $s_k(t)$ satisfies

$$ \lim_{t\to+\infty} \|s_k(t) - s_r(t)\| > 0 \ \ (k \neq r \in \mathcal{M}), $$

and for any $i \in C_k$,

$$ \lim_{t\to+\infty} \|z_i(t) - s_k(t)\| = 0. $$

B. Basic Knowledge

Definition 5 [4] For an integrable function $u(t)$, the fractional-order integral with $t \geqslant 0$ is defined as

$$ {}_0I_t^\beta u(t) = \frac{1}{\Gamma(\beta)} \int_0^t (t-s)^{\beta-1} u(s)\mathrm{d}s, $$

where the order $\beta > 0$ and

$$ \Gamma(\beta) = \int_0^\infty \mathrm{e}^{-t} t^{\beta-1}\mathrm{d}t. $$

Definition 6 [4] For a differentiable function $u(t)$, the Caputo fractional-order derivative with $0 < \beta < 1$ is defined as

$$ {}_0^C D_t^\beta u(t) = \frac{1}{\Gamma(1-\beta)} \int_0^t \frac{u'(s)}{(t-s)^\beta}\mathrm{d}s, \quad t \geqslant 0. $$

Lemma 1 [28] For the differentiable vector-valued function $u(t) \in H^n$, one has

$$ {}_0^C D_t^\beta \big(u^*(t)u(t)\big) \leqslant \big({}_0^C D_t^\beta u^*(t)\big)u(t) + u^*(t){}_0^C D_t^\beta u(t), $$

in which $0 < \beta < 1$.

Lemma 2 [28] For any $z_1, z_2 \in H$, and any real constant $\varepsilon > 0$, the following inequality is true

$$ \bar{z}_1 z_2 + \bar{z}_2 z_1 \leqslant \varepsilon \bar{z}_1 z_1 + \frac{1}{\varepsilon}\bar{z}_2 z_2. $$

Lemma 3 [29] If $H \in \mathcal{H}_1$, then all eigenvalues of the $H - A$ are negative, in which $A = \mathrm{diag}(a_1, a_2, \ldots, a_N)$, a_i is nonnegative constant satisfying $a_1 + a_2 + \cdots + a_N > 0$.

III. Main Result

To realize the cluster synchronization for quaternion-valued FOCCNs, a hybrid control strategy is designed.

First of all, the control scheme $u_i(t)$ in Eq. (2) is designed by

$$ u_i(t) = -d_i\delta_i(t)\Gamma(z_i(t) - s_k(t)), \ \ i \in C_k \tag{4} $$

where if node i is controlled, then $d_i = 1$, otherwise $d_i = 0$. $\delta_i(t)$ is a time-varying feedback gain which satisfies

$$ {}_0^C D_t^\beta \delta_i(t) = d_i\omega_i(z_i(t) - s_k(t))^*\Gamma(z_i(t) - s_k(t)) \tag{5} $$

where $\delta_i(0) \geqslant 0$ and constant $\omega_i > 0$ for $i \in C_k$.

In the k-th cluster, for node $i = \ell_{k-1}+1, \ell_{k-1}+2, \ldots, \ell_k$, let error function $e_i^k(t) = z_i(t) - s_k(t)$ for $i \in C_k$, $k \in \mathcal{M}$. From Definition 3, it follows that $\sum\limits_{r=1}^m \sum\limits_{j\in C_r} H_{ij}s_r(t) = 0$, which derives

$$ \sum_{r=1}^m \sum_{j\in C_r} H_{ij}z_j(t) - \sum_{r=1}^m \sum_{j\in C_r} H_{ij}s_r(t) = \sum_{r=1}^m \sum_{j\in C_r} H_{ij}e_j^r(t). $$

According to FOCCNs in Eqs. (2) and (3), the error system can be depicted as

$$ \begin{aligned} {}_0^C D_t^\beta e_i^k(t) = \tilde{f}(e_i^k(t)) + \varDelta \sum_{r=1}^m \sum_{j\in C_r} H_{ij}\Gamma e_j^r(t) - d_i\delta_i(t)\Gamma e_i^k(t), \\ i \in C_k, \ k \in \mathcal{M} \end{aligned} \tag{6} $$

where $\tilde{f}(e_i^k(t)) = f_k(z_i(t)) - f(s_k(t))$.

For convenience, define

$$ \begin{aligned} &E_k(t) = (e_{\ell_{k-1}+1}^{\mathrm{T}}(t), e_{\ell_{k-1}+2}^{\mathrm{T}}(t), \ldots, e_{\ell_k}^{\mathrm{T}}(t))^{\mathrm{T}}, \\ &\bar{H} = \max_{i\in C_k, j\in C_r, r\neq k}\{|H_{ij}|\}, \\ &\bar{\gamma} = \max_{1\leqslant l\leqslant n}\{\gamma_l\}, \\ &\underline{\gamma} = \min_{1\leqslant l\leqslant n}\{\gamma_l\}, \\ &\mu = \max_{k\in\mathcal{M}}\{N - (\ell_k - \ell_{k-1})\}. \end{aligned} $$

Theorem 1 Assume Assumption 1 holds and the coupling matrix $H \in \mathcal{H}_3$, the cluster synchronization of the quaternion-valued FOCCNs in Eq. (2) under controller in Eqs. (4) and (5) can be achieved with adaptive coupling.

$$ \begin{aligned} {}_0^C D_t^\beta \sigma(t) &= \lambda_{ij}H_{ij}(z_i(t) - z_j(t))^*\Gamma(z_i(t) - z_j(t)), \\ & i, j \in C_k \end{aligned} \tag{7} $$

where $\sigma(0) > 0$ and constant $\lambda_{ij} > 0$.

Proof Construct the Lyapunov function

$$ V(t) = W_1(t) + W_2(t), $$

where

$$ W_1(t) = \frac{1}{2}\sum_{k=1}^m \sum_{i\in C_k} e_i^{k^*}(t)e_i^k(t), $$

$$ \begin{aligned} W_2(t) = &\sum_{k=1}^m \sum_{i\in C_k} \sum_{j\in C_k} \frac{1}{4\lambda_{ij}}(\sigma(t) - \eta)^2 + \\ &\sum_{k=1}^m \sum_{i\in C_k} \frac{1}{2\omega_i}(\delta_i(t) - \eta)^2, \end{aligned} $$

where η is a positive constant.

Based on Lemma 1, calculating the fractional-order derivative of $W_1(t)$, one has

$$ \begin{aligned} &{}_0^C D_t^\beta W_1(t) \leqslant \\ &\frac{1}{2}\sum_{k=1}^m \sum_{i\in C_k}\Big(({}_0^C D_t^\beta e_i^k(t))^* e_i^k(t) + e_i^{k^*}\,({}_0^C D_t^\beta e_i^k(t))\Big) = \\ &\sum_{k=1}^m \sum_{i\in C_k}\Big(-d_i\delta_i(t)e_i^{k^*}(t)\Gamma e_i^k(t) + \\ &\frac{1}{2}(\tilde{f}^*(e_i^k(t))e_i^k(t) + e_i^{k^*}(t)\tilde{f}(e_i^k(t))) + \\ &\frac{\varDelta}{2}\sum_{r=1}^m \sum_{j\in C_r} H_{ij}(e_j^{r*}(t)\Gamma e_i^k(t) + e_i^{k^*}(t)\Gamma e_j^r(t))\Big) \end{aligned} \tag{8} $$

According to Assumption 1, one has

$$\frac{1}{2}\sum_{k=1}^{m}\sum_{i\in C_k}(\tilde{f}^*(e_i^k(t))e_i^k(t)+e_i^{k*}(t)\tilde{f}(e_i^k(t)))\leqslant \frac{L}{2}\sum_{k=1}^{m}\sum_{i\in C_k}e_i^{k*}(t)e_i^k(t) \tag{9}$$

Applying $H_{ij}=H_{ji}$, it can be derived

$$\begin{aligned}&\frac{\Delta}{2}\sum_{k=1}^{m}\sum_{i\in C_k}\sum_{r=1}^{m}\sum_{j\in C_r}H_{ij}(e_j^{r*}(t)\Gamma e_i^k(t)+e_i^{k*}(t)\Gamma e_j^r(t))=\\&\frac{\sigma(t)}{2}\sum_{k=1}^{m}\sum_{i\in C_k}\sum_{j\in C_k}H_{ij}(e_j^{k*}(t)\Gamma e_i^k(t)+e_i^{k*}(t)\Gamma e_j^k(t))+\\&\frac{1}{2}\sum_{k=1}^{m}\sum_{i\in C_k}\sum_{r=1,r\neq k}^{m}\sum_{j\in C_r}H_{ij}(e_j^{r*}(t)\Gamma e_i^k(t)+e_i^{k*}(t)\Gamma e_j^r(t))=\\&\sigma(t)\sum_{k=1}^{m}\sum_{i\in C_k}\sum_{j\in C_k}H_{ij}e_i^{k*}(t)\Gamma e_j^k(t)+\\&\frac{1}{2}\sum_{k=1}^{m}\sum_{i\in C_k}\sum_{r=1,r\neq k}^{m}\sum_{j\in C_r}H_{ij}(e_j^{r*}(t)\Gamma e_i^k(t)+e_i^{k*}(t)\Gamma e_j^r(t))\end{aligned} \tag{10}$$

where

$$\begin{aligned}&\frac{1}{2}\sum_{k=1}^{m}\sum_{i\in C_k}\sum_{r=1,r\neq k}^{m}\sum_{j\in C_r}H_{ij}(e_j^{r*}(t)\Gamma e_i^k(t)+e_i^{k*}(t)\Gamma e_j^r(t))=\\&\frac{1}{2}\sum_{k=1}^{m}\sum_{i\in C_k}\sum_{r=1,r\neq k}^{m}\sum_{j\in C_r}\sum_{l=1}^{n}H_{ij}\gamma_l(\overline{e_{jl}^r(t)}e_{il}^k(t)+\overline{e_{il}^k(t)}e_{jl}^r(t))\leqslant\\&\frac{1}{2}\sum_{k=1}^{m}\sum_{i\in C_k}\sum_{r=1,r\neq k}^{m}\sum_{j\in C_r}\sum_{l=1}^{n}H_{ij}\gamma_l(|e_{il}^k(t)|_2^2+|e_{jl}^r(t)|_2^2)\leqslant\\&\frac{1}{2}\sum_{k=1}^{m}\sum_{i\in C_k}\sum_{r=1,r\neq k}^{m}\sum_{j\in C_r}\sum_{l=1}^{n}\gamma_l|H_{ij}|(|e_{il}^k(t)|_2^2+|e_{jl}^r(t)|_2^2)=\\&\bar{H}\bar{\gamma}\mu\sum_{k=1}^{m}\sum_{i\in C_k}\sum_{l=1}^{n}|e_{il}^k(t)|_2^2.\end{aligned}$$

Hence, Eq. (10) can be rewritten as

$$\begin{aligned}&\frac{\Delta}{2}\sum_{k=1}^{m}\sum_{i\in C_k}\sum_{r=1}^{m}\sum_{j\in C_r}H_{ij}(e_j^{r*}(t)\Gamma e_i^k(t)+e_i^{k*}(t)\Gamma e_j^r(t))\leqslant\\&\sigma(t)\sum_{k=1}^{m}\sum_{i\in C_k}\sum_{j\in C_k}H_{ij}e_i^{k*}(t)\Gamma e_j^k(t)+\\&\bar{H}\bar{\gamma}\mu\sum_{k=1}^{m}\sum_{i\in C_k}\sum_{l=1}^{n}|e_{il}^k(t)|_2^2\end{aligned} \tag{11}$$

Substituting Eqs. (9)–(11) into Eq. (8), it follows

$$\begin{aligned}&{}_0^C D_t^\beta W_1(t)\leqslant\\&\sum_{k=1}^{m}\sum_{i\in C_k}e_i^{k*}(t)\Big(\frac{L}{2}+\bar{H}\bar{\gamma}\mu-d_i\delta_i(t)\Gamma\Big)I_n e_i^k(t)+\\&\sigma(t)\sum_{k=1}^{m}\sum_{i\in C_k}\sum_{j\in C_k}H_{ij}e_i^{k*}(t)\Gamma e_j^k(t)\end{aligned} \tag{12}$$

On the other hand, we have

$$\begin{aligned}&{}_0^C D_t^\beta W_2(t)\leqslant\\&\sum_{k=1}^{m}\sum_{i\in C_k}\sum_{j\in C_k}\frac{H_{ij}}{2}(\sigma(t)-\eta)(z_i(t)-z_j(t))^*\Gamma(z_i(t)-z_j(t))+\\&\sum_{k=1}^{m}\sum_{i\in C_k}(\delta_i(t)-\eta)d_i e_i^{k*}(t)\Gamma e_i^k(t)=\\&-\sum_{k=1}^{m}\sum_{i\in C_k}\sum_{j\in C_k}H_{ij}(\sigma(t)-\eta)(e_i^{k*}(t)\Gamma e_j^k(t))+\\&\sum_{k=1}^{m}\sum_{i\in C_k}(\delta_i(t)-\eta)d_i e_i^{k*}(t)\Gamma e_i^k(t)\end{aligned} \tag{13}$$

Together with Eqs. (12) and (13), one can obtain

$$\begin{aligned}&{}_0^C D_t^\beta V(t)\leqslant\\&\sum_{k=1}^{m}\sum_{i\in C_k}e_i^{k*}(t)\Big(\Big(\frac{L}{2}+\bar{H}\bar{\gamma}\mu\Big)\otimes I_n\Big)e_i^k(t)+\\&\eta\sum_{k=1}^{m}\sum_{i,j\in C_k}H_{ij}(e_i^{k*}(t)\Gamma e_j^k(t))-\\&\eta\sum_{k=1}^{m}\sum_{i\in C_k}d_i e_i^{k*}(t)\Gamma e_i^k(t)\leqslant\\&\sum_{k=1}^{m}\sum_{i\in C_k}e_i^{k*}(t)\Big(\Big(\frac{L}{2}+\bar{H}\bar{\gamma}\mu\Big)\otimes I_n\Big)e_i^k(t)+\\&\eta\underline{\gamma}\sum_{k=1}^{m}E_k^*(t)(H_{kk}-I_{\ell_k})E_k(t),\end{aligned}$$

where $\otimes$ represents the Kronecker product.

Based on Lemma 3, it can be derived that $H_{kk}-I_{\ell_k}$ is a negative definite matrix. Then, one can select a suitable η satisfying

$${}_0^C D_t^\beta V(t)\leqslant-\sum_{k=1}^{m}E_k^*(t)E_k(t)=-2W_1(t) \tag{14}$$

From the definition of $W_1(t)$ and Eq. (14), one can find constant $M>0$ such that $|{}_{t_0}^C D_t^\beta W_1(t)|\leqslant M$. Then, for any $\varepsilon>0$ and $0\leqslant t_1<t_2$, we have

$$\begin{aligned}&|W_1(t_2)-W_1(t_1)|=\\&|{}_0I_{t_2}^\beta\, {}_0^C D_{t_2}^\beta W_1(t_2)-{}_0I_{t_1}^\beta\, {}_0^C D_{t_1}^\beta W_1(t_1)|\leqslant\\&\frac{1}{\Gamma(\beta)}\int_0^{t_1}[(t_1-s)^{\beta-1}-(t_2-s)^{\beta-1}]|{}_0^C D_s^\beta W_1(s)|\mathrm{d}s+\\&\frac{1}{\Gamma(\beta)}\int_{t_1}^{t_2}(t_1-s)^{\beta-1}|{}_0^C D_s^\beta W_1(s)|\mathrm{d}s\leqslant\\&\frac{M}{\Gamma(\beta+1)}[(t_1)^\beta-(t_2)^\beta+2(t_2-t_1)^\beta]\leqslant\\&\frac{2M}{\Gamma(\beta+1)}(t_2-t_1)^\beta<\varepsilon,\end{aligned}$$

where $|t_2-t_1|<\delta(\varepsilon)=(\varepsilon\Gamma(\beta+1)/2M)^{\frac{1}{\beta}}$. As δ is free of t_1 and t_2, $W_1(t)$ is uniformly continuous.

In the following, $\lim_{t\to+\infty}W_1(t)=0$ will be proved.

Assume for all $T>0$ and constant $\varepsilon_0>0$, there exists $t>T$

such that $W_1(t) > \varepsilon_0$. Then, one can find an infinite sequence of t_s satisfying $t_1 < t_2 < \cdots < t_s < t_{s+1} < \cdots$ and $t_s \to \infty$ as $k \to \infty$ leads to $W_1(t_s) > \varepsilon_0$. Since $W_1(t)$ is uniformly continuous, the constant $\eta > 0$ exists such that for t' and t'', one has $|t' - t''| < \eta$, so $|W_1(t') - W_1(t'')| < \varepsilon_0/2$, implying that $W_1(t) > W_1(t_s)/2 > \varepsilon_0/2$ for $|t - t_s| < \eta$, $s = 1, 2, \ldots$.

Without loss of generality, it is assumed that these intervals are disjoint and $t_1 - \eta > 0$, which means $t_s - \eta < t_s + \eta < t_{s+1} - \eta$ and $t_{s+1} - t_s > 2\eta$.

Making fractional integration from 0 to $t_s + \eta$ on both sides of Eq. (14), and in view of Lagrange formula, Eq. (15) can be derived

$$
\begin{aligned}
&V(t_s+\eta) - V(0) \leqslant -\frac{2}{\Gamma(\beta)} \int_0^{t_s+\eta} (t_s+\eta-s)^{\beta-1} W_1(\tau)\mathrm{d}\tau, \\
&V(0) - V(t_s+\eta) \geqslant \frac{\varepsilon_0}{\Gamma(\beta)} \int_0^{t_s+\eta} (t_s+\eta-\tau)^{\beta-1}\mathrm{d}\tau \geqslant \\
&\frac{\varepsilon_0}{\Gamma(\beta)} \sum_{i=1}^{s} \int_{t_i-\eta}^{t_i+\eta} (t_s+\eta-\tau)^{\beta-1}\mathrm{d}\tau = \\
&\frac{\varepsilon_0}{\Gamma(\beta+1)} \sum_{i=1}^{s} [(t_s - t_i + 2\eta)^{\beta} - (t_s - t_i)^{\beta}] = \\
&\frac{\varepsilon_0}{\Gamma(\beta+1)} \sum_{i=1}^{s} [\beta\,(t_s - t_i + 2\theta\eta)^{\beta-1}\, 2\eta] = \\
&\frac{2\varepsilon_0\,\eta}{\Gamma(\beta)} \sum_{i=1}^{s} (t_s - t_i + 2\theta\eta)^{\beta-1}
\end{aligned} \tag{15}
$$

where $0 < \theta < 1$.

For $s = 1, 2, \ldots$, denote $\omega = \sup\{t_{s+1} - t_s\}$. It is easy to obtain that $2\eta \leqslant \omega$. Then,

$$
\begin{aligned}
&(t_s - t_i + 2\theta\eta)^{1-\beta} \leqslant \\
&\sum_{r=i+1}^{s} (t_r - t_{r-1})^{1-\beta} + (2\theta\eta)^{1-\beta} \leqslant \\
&(s-i)\omega^{1-\beta} + (\theta\omega)^{1-\beta} \leqslant \\
&(s-i+1)\omega^{1-\beta},
\end{aligned}
$$

combining with Eq. (15), it follows

$$
V(0) - V(t_s+\eta) \geqslant \frac{2\varepsilon_0\,\eta}{\Gamma(\beta)} \omega^{\beta-1} \sum_{i=1}^{s} \frac{1}{i} \tag{16}
$$

Based on Eq. (16), as $s \to +\infty$, $V(t_s+\eta) \to -\infty$, which contradicts $V(t) \geqslant 0$. Hence, $\lim_{t\to+\infty} W_1(t) = 0$ holds.

It can be concluded from the above, under controller in Eqs. (4) and (5), the system in Eq. (2) is asymptotically synchronized to $s_k(t)$ with adaptive law in Eq. (7). ∎

Remark 1 Cluster synchronization of quaternion-valued FOCCNs with nonidentical nodes is investigated in Theorem 1. Due to the fact that $V(t)$ is made up of $W_1(t)$ and $W_2(t)$, the fractional-order Lyapunov direct method is not suitable for this case. The mean value theorem and the reduction to absurdity are used to achieve synchronization of FOCCNs.

Next, we consider the synchronization of quaternion-valued FOCCNs with identical nodes.

In Eq. (2), if $f_k(z_i(t)) = f(z_i(t))$, $k \in \mathcal{M}$, the FOCNs in Eq. (2) can be described as

$$
{}_0^C D_t^{\beta} z_i(t) = f(z_i(t)) + \sigma(t) \sum_{j=1}^{N} H_{ij} \Gamma z_j(t) + u_i(t), \quad i \in C_k \tag{17}
$$

then all the states of nodes in Eq. (17) can be synchronized onto the manifold $z_1(t) = z_2(t) = \cdots = z_N(t) = s(t)$.

The dynamic of $s(t)$ can be depicted as

$$
{}_0^C D_t^{\beta} s(t) = f(s(t)) \tag{18}
$$

The cluster synchronization of FOCNs in Eq. (2) is reduced to the synchronization of quaternion-valued FOCNs with identical nodes. The control scheme in Eq. (4) is designed by

$$
u_i(t) = -\delta_i(t)\Gamma(z_i(t) - s(t)), \;\; i \in \mathcal{N} \tag{19}
$$

where $\delta_i(t)$ is a feedback gain, which is constructed by

$$
{}_0^C D_t^{\beta} \delta_i(t) = \omega_i (z_i(t) - s(t))^* \Gamma(z_i(t) - s(t)), \;\; i \in \mathcal{N} \tag{20}
$$

where $\omega_i > 0$.

Corollary 1 Suppose Assumption 1 holds and $H \in \mathcal{H}_1$, the quaternion-valued FOCNs under controller in Eqs. (19) and (20) are asymptotically synchronized to the desired state in Eq. (18), if the coupling strength $\sigma(t)$ has the following form

$$
\begin{aligned}
{}_0^C D_t^{\beta} \sigma(t) = &\lambda_{ij} H_{ij} (z_i(t) - z_j(t))^* \Gamma(z_i(t) - z_j(t)), \\
&i, j \in \mathcal{N}
\end{aligned} \tag{21}
$$

where $\lambda_{ij} > 0$.

Remark 2 In Refs. [11, 16, 19–22, 30], synchronization of network systems with identical nodes has been studied. As a matter of fact, it is unpracticable that all nodes share the common dynamical behavior, such as grouping of opinion dynamics in social network and grouping of colony in biological network. To further explain these interesting phenomena, cluster synchronization is proposed for FOCCNs, in which the nodes within the same community own consistent behaviors, otherwise, they exhibit different dynamic behaviors.

Remark 3 In Ref. [11], by using the local information of the network and adjusting the coupling weights, synchronization for FOCNs is realized without external forces. In Refs. [16, 22], by adding feedback controller on the network with constant coupling strength, synchronization of FOCNs is achieved. Different from these, time-varying coupling strength and feedback control gain are automatically adjusted with the evolution of states to guarantee synchronization of FOCCNs. It is of great significance to not only study the synchronization of FOCNs, but also understand the dynamic contract and collaboration in neural network interactions.

IV. Illustrative Example

Consider quaternion-valued FOCCN with two clusters, which is depicted by

$$
\begin{aligned}
{}_0^C D_t^{\beta} z_i(t) = f_k(z_i(t)) + \sigma(t) \sum_{j=1}^{N} H_{ij} \Gamma z_j(t) + u_i(t), \\
i = 1, 2, \ldots, 7
\end{aligned} \tag{22}
$$

where $\beta = 0.97$, $z_i(t) = (z_{i1}(t), z_{i2}(t))^{\mathrm{T}}$, $C_1 = \{1,2,3\}$, $C_2 = \{4,5,6,7\}$, $k = 1,2$, $\Gamma = I_3$, and

$$f_1(z_i(t)) = Cz_i(t) + Ag_1(z_i(t)),$$
$$f_2(z_i(t)) = Cz_i(t) + Bg_2(z_i(t)),$$

where

$$g_1(z_i(t)) = \tanh(z_i^{\mathrm{R}}) + \tanh(z_i^{\mathrm{I}})\imath + \tanh(z_i^{\mathrm{J}})\jmath + \tanh(z_i^{\mathrm{K}})k,$$

$$g_2(z_i(t)) = 0.1\frac{1-\exp(-z_i^{\mathrm{R}}(t))}{1+\exp(-z_i^{\mathrm{R}}(t))} + \tanh(z_i^{\mathrm{I}})\imath +$$
$$0.1\frac{1-\exp(-z_i^{\mathrm{J}}(t))}{1+\exp(-z_i^{\mathrm{J}}(t))}\jmath + \tanh(z_i^{\mathrm{K}})k,$$

and

$$C = \mathrm{diag}(c_1, c_2) = \begin{pmatrix} -1.9 & 0.0 \\ 0.0 & -1.9 \end{pmatrix},$$

$$A = \begin{pmatrix} 2.00-2.25\imath+1.70\jmath+k & -0.80+0.89\imath-0.32\jmath-0.78k \\ -1.67-1.99\imath-1.50\jmath-3.27k & 1.00+1.77\imath+0.98\jmath+k \end{pmatrix},$$

$$B = \begin{pmatrix} 2.30-2.10\imath+2.40\jmath+1.50k & -1.20-2.90\imath-0.65\jmath-k \\ -1.50-1.90\imath-\jmath-2.60k & 1.00-2.80\imath+0.75\jmath+0.88k \end{pmatrix}.$$

The communication topology structure of FOCCNs in Eq. (22) is depicted in Fig. 1. The dynamical behavior of desired cluster synchronization states of FOCCNs in Eq. (22) is given by

$$ {}_0^C D_t^{\beta} s_k(t) = f_k(s(t)), \quad k = 1,2 \tag{23}$$

with initial values $s_1(0) = (-0.50+0.20\imath-0.70\jmath+0.50k, 0.60-0.10\imath-0.50\jmath-0.20k)^{\mathrm{T}}$ and $s_2(0) = (0.30-0.58\imath+0.50\jmath-0.40k, 1.00-0.40\imath+0.50\jmath+k)^{\mathrm{T}}$.

In the following, under controller in Eqs. (4) and (5), we consider the cluster synchronization with adaptive coupling strength in Eq. (7). Choose $\sigma(0) = 0.15$, $\lambda_{ij} = 2$, and $\omega_i = 0.3$ for $i, j \in C_k$. Randomly select the real and imaginary parts of the initial value of FOCCNs in Eq. (16) from $[-1,1]$. In Eq. (4), the initial values of feedback control gain are randomly chosen from $[0.00, 0.25]$. Based on Theorem 1, the system in Eq. (22) with the adaptive couple strength in Eq. (7) is synchronized to the desired orbit in Eq. (23) under controller in Eq. (4), which is shown in Figs. 2–5. The time evolutions of synchronization error are depicted in Figs. 6–9. The trajectories of time-varying coupling $\sigma(t)$ and feedback gains $\delta_i(t)$ are demonstrated in Figs. 10 and 11.

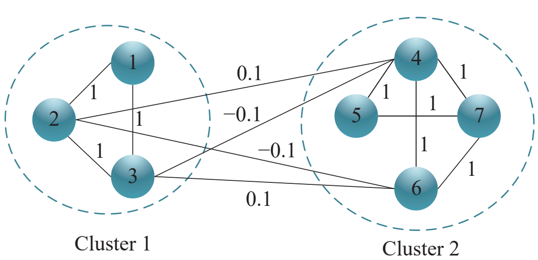


Figure 1 Communication topology structure of FOCCNs in Eq. (22).

V. Conclusion

In this paper, cluster synchronization of quaternion-valued FOCCNs with nonidentical nodes is studied. First, considering the interaction of nodes in the cluster and the influence of external control, the hybrid controller is applied to FOCCNs, that is, the adaptive feedback control gains and adaptive coupling strength are considered simultaneously. Subsequently, the mean value theorem and the proof by contradiction are used to prove cluster synchronization of quaternion-valued FOCCNs. Then, a case analysis approach about synchronization of FOCNs with identical nodes is also presented, which provides more flexibility to analyze fractional-order systems. Finally, a numerical example is presented to show the effectiveness of theoretical results.

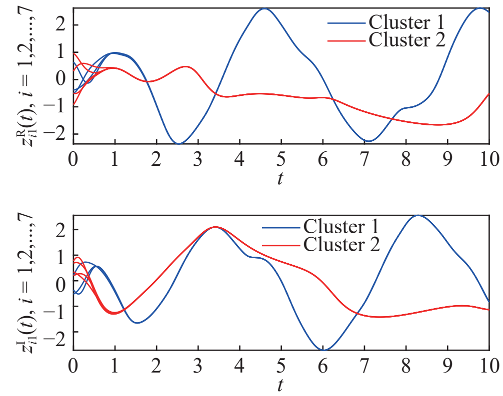


Figure 2 Evolution of $z_{i1}^{\mathrm{R}}(t)$ and $z_{i1}^{\mathrm{I}}(t)$ in FOCCNs under adaptive control.

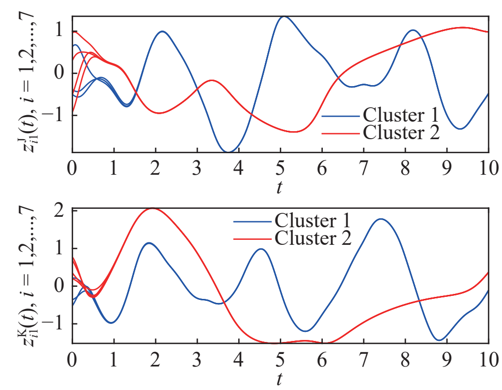


Figure 3 Evolution of $z_{i1}^{\mathrm{J}}(t)$ and $z_{i1}^{\mathrm{K}}(t)$ in FOCCNs under adaptive control.

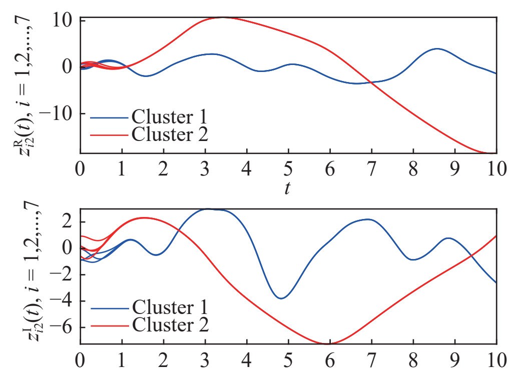


Figure 4 Evolution of $z_{i2}^{\mathrm{R}}(t)$ and $z_{i2}^{\mathrm{I}}(t)$ in FOCCNs under adaptive control.

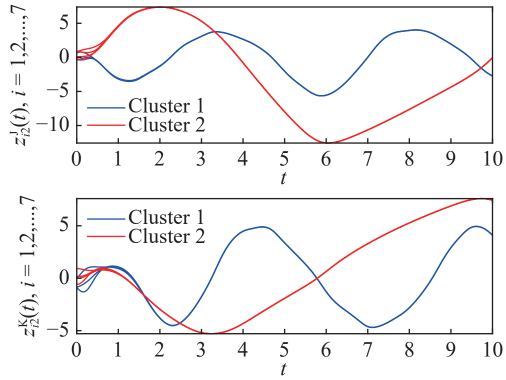


Figure 5 Evolution of $z_{i2}^{\mathrm{J}}(t)$ and $z_{i2}^{\mathrm{K}}(t)$ in FOCCNs under adaptive control.

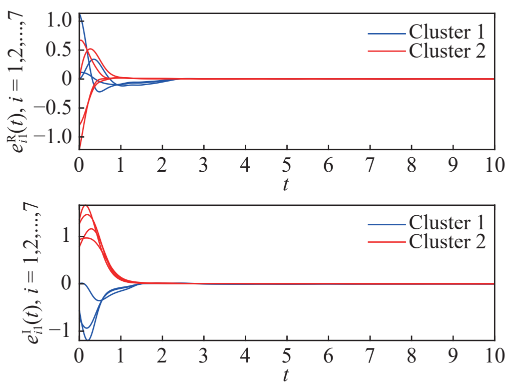


Figure 6 Time evolution of synchronization errors $e_{i1}^{\mathrm{R}}(t)$ and $e_{i1}^{\mathrm{I}}(t)$.

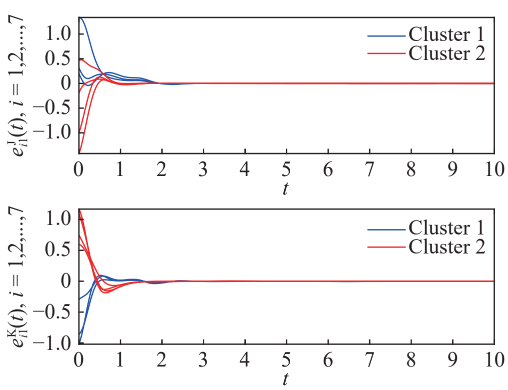


Figure 7 Time evolution of synchronization errors $e_{i1}^{\mathrm{J}}(t)$ and $e_{i1}^{\mathrm{K}}(t)$.

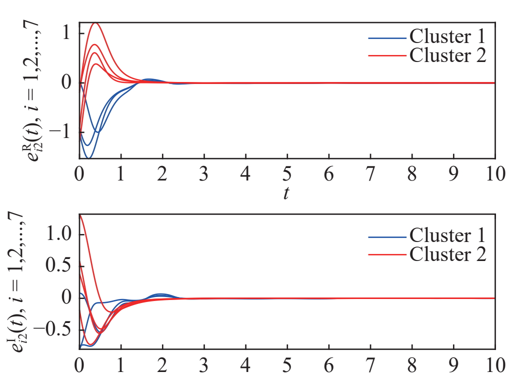


Figure 8 Time evolution of synchronization errors $e_{i2}^{\mathrm{R}}(t)$ and $e_{i2}^{\mathrm{I}}(t)$.

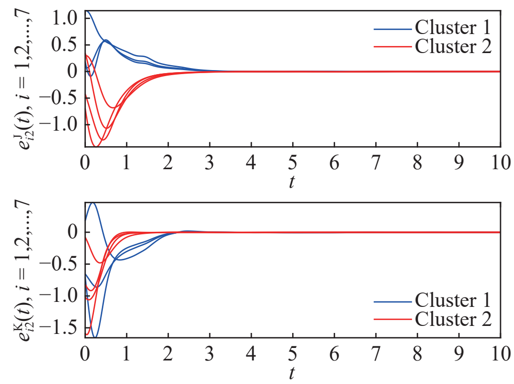


Figure 9 Time evolution of synchronization errors $e_{i2}^{\mathrm{J}}(t)$ and $e_{i2}^{\mathrm{K}}(t)$.

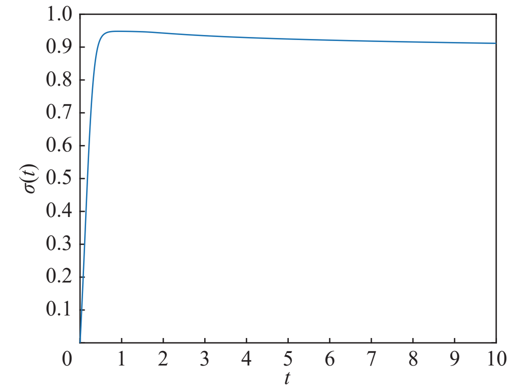


Figure 10 Evolution of adaptive coupling strength $\sigma(t)$.

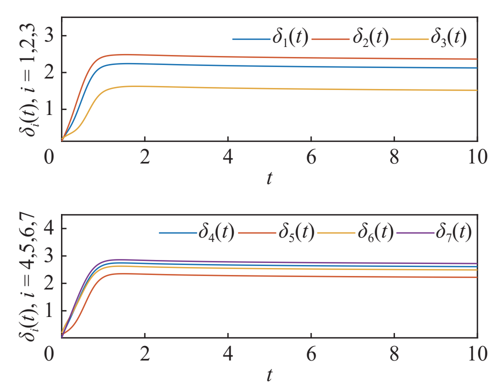


Figure 11 Evolution of control gain $\delta_i(t)$.

Acknowledgment

This work was supported by the National Natural Science Foundation of China (No. 62373089), the Synthetical Automation for Process Industries (SAPI) Fundamental Research Funds (No. 2018ZCX22), and the Natural Science Foundation of Liaoning Province, China (No. 2022JH25/10100008).

References

[1] S. H. Strogatz, Exploring complex networks, *Nature*, 2001, 410(6825), 268–276.

[2] S. Arianos, E. Bompard, A. Carbone, and F. Xue, Power grid vulnerability: a complex network approach, *Chaos*, 2008, 19(1), 013119.

[3] H. Izadpanah, A millimeter wave broadband wireless access technology demonstrator for the next generation internet network reach extension, *IEEE Commun. Mag*., 2001, 39(9), 140–145.

[4] I. Podlubny, Fractional differential equations, *Math. Sci. Eng*., 1999, 198, 1–340.

[5] A. Ahmadian, F. Ismail, S. Salahshour, D. Baleanu, and F. Ghaemi, Uncertain viscoelastic models with fractional order: a new spectral Tau method to study the numerical simulations of the solution, *Commun. Nonlinear Sci. Numer. Simul*., 2017, 53, 44–64.

[6] J. Y. Zhou, Y. Ye, A. Arenas, S. Gómez, and Y. Zhao, Pattern formation and bifurcation analysis of delay induced fractional-order epidemic spreading on networks, *Chaos Solitons Fractals*, 2023, 174, 113805.

[7] J. G. Qu, Q. W. Zhang, A. M. Yang, Y. M. Chen, and Q. Zhang, Variational fractional-order modeling of viscoelastic axially moving plates and vibration simulation, *Commun. Nonlinear Sci. Numer. Simul*., 2024, 130, 107707.

[8] G. M. Mahmoud, M. E. Ahmed, and T. M. Abed-Elhameed, On fractional-order hyperchaotic complex systems and their generalized function projective combination synchronization, *Optik*, 2017, 130, 398–406.

[9] K. M. Babanli and R. Ortac Kabaoglu, Synchronization of fuzzy-chaotic systems with Z-controller in secure communication, *Inf. Sci*., 2024, 657, 119988.

[10] L. Přibylová, D. Sen, and V. Eclerová, Destabilization of a seasonal synchronization in a population model with a seasonally varying Allee effect, *Appl. Math. Comput*., 2024, 462, 128331.

[11] B. B. Zheng and Z. S. Wang, Adaptive synchronization of fractional-order complex-valued coupled neural networks via direct error method, *Neurocomputing*, 2022, 486, 114–122.

[12] J. Q. Wang and X. W. Liu, Cluster synchronization for multiweighted and directed fractional-order networks with cooperative-competitive interactions, *IEEE Trans. Circuits Syst. II: Express Briefs*, 2022, 69(11), 4359–4363.

[13] X. L. Yuan, G. J. Ren, H. Wang, and Y. G. Yu, Mean-square synchronization of fractional-order stochastic complex network via pinning control, *Neurocomputing*, 2022, 513, 153–164.

[14] W. Chen, Y. G. Yu, X. D. Hai, and G. J. Ren, Adaptive quasi-synchronization control of heterogeneous fractional-order coupled neural networks with reaction-diffusion, *Appl. Math. Comput*., 2022, 427, 127145.

[15] F. F. Du, J. G. Lu, and Q. H. Zhang, Delay-dependent finite-time synchronization criterion of fractional-order delayed complex networks, *Commun. Nonlinear Sci. Numer. Simul*., 2023, 119, 107072.

[16] S. Yang, C. Hu, J. Yu, and H. J. Jiang, Finite-time cluster synchronization in complex-variable networks with fractional-order and nonlinear coupling, *Neural Netw*., 2021, 135, 212–224.

[17] M. Mazza, G. Cola, and M. Tesconi, Modularity-based approach for tracking communities in dynamic social networks, *Knowl.-Based Syst*., 2023, 281, 111067.

[18] X. W. Liu and T. P. Chen, Finite-time and fixed-time cluster synchronization with or without pinning control, *IEEE Trans. Cybern*., 2018, 48(1), 240–252.

[19] P. Selvaraj, O. M. Kwon, S. H. Lee, and R. Sakthivel, Cluster synchronization of fractional-order complex networks via uncertainty and disturbance estimator-based modified repetitive control, *J. Franklin Inst*., 2021, 358(18), 9951–9974.

[20] P. Liu, M. L. Xu, J. W. Sun, and S. P. Wen, Cluster synchronization of multiple fractional-order recurrent neural networks with time-varying delays, *IEEE Trans. Neural Netw. Learn. Syst*., 2023, 34(8), 4007–4018.

[21] Q. K. Kang, Q. X. Yang, Z. L. Lin, and Q. T. Gan, Finite-time cluster synchronization of delayed fractional-order fully complex-valued community networks, *IEEE Access*, 2022, 10, 103948–103962.

[22] P. Liu, Z. G. Zeng, and J. Wang, Asymptotic and finite-time cluster synchronization of coupled fractional-order neural networks with time delay, *IEEE Trans. Neural Netw. Learn. Syst*., 2020, 31(11), 4956–4967.

[23] J. L. Wang, Z. Qin, H. N. Wu, and T. W. Huang, Finite-time synchronization and H_∞ synchronization of multiweighted complex networks with adaptive state couplings, *IEEE Trans. Cybern*., 2020, 50(2), 600–612.

[24] S. B. Zhu, J. Zhou, Q. X. Zhu, N. Li, and J. A. Lu, Adaptive exponential synchronization of complex networks with nondifferentiable time-varying delay, *IEEE Trans. Neural Netw. Learn. Syst*., 2023, 34(10), 8124–8130.

[25] T. Isokawa, T. Kusakabe, N. Matsui, and F. Peper, Quaternion neural network and its application, in *Proc. 7th International Conference on Knowledge-Based Intelligent Information*, Oxford, UK, 2003, 318–324.

[26] N. Matsui, T. Isokawa, H. Kusamichi, F. Peper, and H. Nishimura, Quaternion neural network with geometrical operators, *J. Intell. Fuzzy Syst.: Apll. Eng. Technol*., 2004, 15(3–4), 149–164.

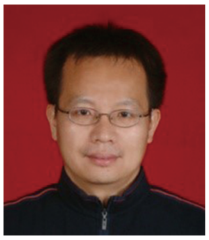
[27] J. Y. Xiao, J. D. Cao, J. Cheng, S. P. Wen, R. M. Zhang, and S. M. Zhong, Novel inequalities to global Mittag-Leffler synchronization and stability analysis of fractional-order quaternion-valued neural networks, *IEEE Trans. Neural Netw. Learn. Syst*., 2021, 32(8), 3700–3709.

[28] H. L. Li, H. J. Jiang, and J. D. Cao, Global synchronization of fractional-order quaternion-valued neural networks with leakage and discrete delays, *Neurocomputing*, 2020, 385, 211–219.

[29] C. Hu, J. Yu, H. J. Jiang, and Z. D. Teng, Pinning synchronization of weighted complex networks with variable delays and adaptive coupling weights, *Nonlinear Dyn*., 2012, 67(2), 1373–1385.

[30] C. Hu, H. B. He, and H. J. Jiang, Synchronization of complex-valued dynamic networks with intermittently adaptive coupling: a direct error method, *Automatica*, 2020, 112, 108675.

Bibo Zheng received the MS degree in applied mathematics from College of Mathematics and System Science, Xinjiang University, Urumqi, China, in 2020. She is currently pursuing the PhD degree at College of Information Science and Engineering, Northeastern University, Shenyang, China. Her research interests include fractional-order systems and control theory.



Zhanshan Wang received the BS degree in industry electric automation from Inner Mongolia University of Science & Technology, Baotou, China, in 1994, the MS degree in control theory and control engineering from Liaoning Petrochemical University, Fushun, China, in 2001, and the PhD degree in control theory and control engineering from Northeastern University, Shenyang, China, in 2006. He is currently a professor at Northeastern University, China. His research interests include stability theory of dynamical systems with delays, neural networks theory, complex networks, learning control, fault diagnosis, fault tolerant control, intelligent automation, and their applications in power systems and smart grid.