Event-Based Cooperative Control for Uncertain Multiagent System Using Parallel Adaptive Dynamic Programming

Shanshan Jiao, Qinglai Wei, Fei Dai, Jianchao Wu, Miaosheng Qiu, Bin Zhang, Yongjin Luo, Kunxin Huang, and Genpo Ma

Abstract—**This study explores a new robust consensus control strategy for uncertain multiagent systems and provides an event-based solution to adaptive dynamic programming (ADP) based optimal control. Rather than the control function, the feedback system established symmetrical to the physical system allows the optimal consensus control issue to be handled by the optimal control protocol of an augmented affine system. The feedback system focuses on an auxiliary variable formed in light of the optimality principle and the virtual control input built on a critic neural network (NN). Analysis reveals that the auxiliary variable benefits from decreasing the influence of uncertainty on control performance, while the proposed approach is implemented with fewer communication resources since the critic NN is updated as events occur. Finally, evidence from simulation findings validates the theoretical results.**

Index Terms—**Optimal consensus control, event-based control, robust control, parallel control, adaptive dynamic programming**

I. Introduction

Human comprehension of the world is constantly advancing due to the rapid development of computer networks, virtual reality, and other techniques. Processing systems progressively evolve from single mechanical equipment to high-tech fields, such as aerospace systems composed of several subsystems. As a result, the cooperative control of subsystems to perform tasks has become a topic of interest [1]. Research on consensus control, a classic cooperative control approach, has likewise sprouted up [2, 3].

In Ref. [4], to clarify the settling time, the fixed-time and prescribed-time consensus controls of multiagent systems (MASs) were discussed, and their properties were provided. In Ref. [5], the consensus issue was settled as the stability problem of a single system, and the uncertain parameters were compensated by the dynamic gain, such that the control protocol enabled agents to achieve the full-state consensus. A hybrid system approach was developed for data-based MASs in Ref. [6], and its characteristics laid in an extra reset control part for better transient consensus performance.

However, the impact of uncertainty on system performance cannot be underestimated, further stimulating the development of robust consensus control. In Ref. [7], a quadratic objective function was developed for consensus control of linear MASs, and the condition and weighting matrices were specified to design a distributed consensus controller to settle the coupling. In Ref. [8], a terminal sliding mode control scheme was identified for the consensus of MASs with inherent disturbances, in which the feedback gain was determined by the settling time. A performance-oriented protocol was investigated in Ref. [9] for the desirable system steady state and tunable transient performance in consensus control. Moreover, some finite-time H_∞ consensus criteria generated from the proposed state feedback controller and the adaptive controller were identified in Ref. [10].

The ongoing advancement in emerging fields has also led to the quality requirements for control systems being updated in practical applications, and the trend toward developing a stable consensus control rule that is capable of optimizing the specific index is inevitable [11, 12]. In Ref. [13], the optimal consensus control challenge of MASs was treated as a nonzero-sum game with players denoting agents, and a data-based algorithm was proposed using offline system interaction datasets to ensure the performance of learning algorithms. In Ref. [14], the solution to the optimal consensus control of strict feedback MAS was built on the virtual and actual control rules, and the negative gradient of simple positive function allowed the control rule to be updated. In addition, the parallel control technique [15], as a powerful platform, was employed in Ref. [16], which enabled the consensus control challenge to be settled in virtual space, thereby attaching the consensus of all agents by a unique feedback system.

Subsequently, new insights into the solution of optimal consensus control for uncertain MASs are also inspired. In Ref. [17], a compensator for the dynamic uncertainties was formed using the Lyapunov redesign technique, resulting in a

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Shanshan Jiao is with Institute of Systems Engineering, Macau University of Science and Technology, Macao 999078, China (e-mail: 2202853ymi30001@student.must.edu.mo).

Qinglai Wei is with State Key Laboratory of Multimodal Artificial Intelligence Systems, Institute of Automation, Chinese Academy of Sciences, Beijing 100190, China, and also with School of Artificial Intelligence, University of Chinese Academy of Sciences, Beijing 100049, China (e-mail: qinglai.wei@ia.ac.cn).

Fei Dai, Jianchao Wu, Miaosheng Qiu, Bin Zhang, Yongjin Luo, Kunxin Huang, and Genpo Ma are with Guangzhou Goaland Energy Conservation Tech. Co., Ltd., Guangzhou 510663, China (e-mail: daif@goaland.com.cn; wujc@goaland.com.cn; qiums@goaland.com.cn; zhangb@goaland.com.cn; luoyj@goaland.com.cn; huangkx@goaland.com.cn; magp@goaland.com.cn).

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fully distributed algorithm for optimal consensus control with partial information. The global information of the graph and parameter uncertainty is necessary for the distributed robust optimal control strategy involved in Ref. [18]. In Ref. [19], a distributed optimal cooperative control rule was established with the robustness of MASs guaranteed by the integral sliding mode control method. In Ref. [20], an optimal consensus control approach was analyzed using reinforcement learning to mitigate uncertainty and external disturbances, thereby balancing the control resources and system performance.

Most of the existing robust optimal consensus control methods require compensation for uncertainty or to achieve robust control with the modified performance indicators. The former complicates the robust optimal consensus control, and the latter fails to balance optimality and robustness. Then, the popularity of parallel control motivates us to construct an auxiliary variable to fill this gap, as the strategy shown in Ref. [21].

Moreover, as another performance indicator, the occupation of communication resources in the control field is gradually attracting attention [22, 23]. An event-based robust optimal consensus control strategy was explored in Ref. [24], in which a modified cost function was invoked to guarantee the robustness of uncertain MASs. An event-based controller was used for the position control of the system, and a dual-loop structure multi-rotor unmanned aerial vehicle (UAV) swarm control model was constructed in Ref. [25].

For this reason, we attempt to implement the event-triggered robust optimal consensus control via parallel control. The significant contributions of this paper, in terms of the practical work, are as follows:

(1) An auxiliary variable and a virtual control law are applied to establish the feedback system. This allows the robust optimal consensus control issue of the uncertain MAS to be addressed through an affine augmented system, which takes the virtual control law as the control input and the physical system is treated as its internal dynamic.

(2) The coupled Hamilton-Jacobi (HJ) equations are built for the augmented system of the uncertain and nominal systems, so that a certain unique condition can be determined in light of the equivalence between the HJ equations, thus constituting the auxiliary variable employed in the feedback system.

(3) Different from the common triggered control strategies, invoking a nonzero function enables the update frequency of the control law and the estimation weights of critic neural network (NN) to be further decreased, resulting in robust optimal consensus control online in a more communication resource-efficient way.

The rest of this paper is organized as follows. Section II describes the problem involved in this paper. Event-triggered robust optimal control is detailed in Section III. In Section IV, the proposed method is examined by a numerical example. Section V concludes this study.

Notation $\|\cdot\|$ is the norm. $\lambda_{\min}(\cdot)$ signifies the minimal eigenvalue of the matrix. n-dimensional vectors and $n\times m$-dimensional matrices are denoted as $\mathbb{R}^n$ and $\mathbb{R}^{n\times m}$. 0_n and 1_n represent n-dimensional vectors with 0 and 1, respectively. $(\cdot)^*$ denotes the optimal value of $(\cdot)$. $V_{(\cdot)} = \partial V/\partial(\cdot)$.

II. Problem Formulation

This section provides an overview of graph theory and introduces the robust optimal parallel consensus control issue.

A. Algebraic Graph Theory

$\mathcal{G} \overset{\Delta}{=} (\mathcal{V},\mathcal{E},\mathcal{A})$ draws a graph that covers the interaction between agents in MASs, where $\mathcal{V} = \{v_i, i = 1,2,\ldots,N\}$ shows a set with N agents, the set of edges is $\mathcal{E} \subseteq \mathcal{V}\times\mathcal{V}$, the neighbor set of agent i is $\mathcal{N}_i = \{j \in \mathcal{V} : (j,i) \in \mathcal{E}\}$, and (j,i) demonstrates that the information is transmitted from agent j to agent i. $(j,i) \in \mathcal{E}$ yields the adjacency matrix $\mathcal{A} = \left[a_{ij}\right]$ with $a_{ij} = 1$ and $a_{ii} = 0$. Next, $d_i = \sum_{j\in\mathcal{N}_i} a_{ij}$ forms the in-degree matrix $\mathcal{D} = \mathrm{diag}\{d_1,d_2,\ldots,d_N\}$ to calculate the Laplace matrix $L = \mathcal{D} - \mathcal{A} = \left[l_{ij}\right]$, where $l_{ij} = -a_{ij}$ and $l_{ii} = d_i$. Furthermore, $b_i = 1$ indicates that there exists an interaction between the leader and the i-th follower, and $\mathcal{B} = \mathrm{diag}\{b_1,b_2,\ldots,b_N\}$.

B. Problem Description

The nonlinear MAS with N followers is given as

$$\dot{x}_i = f(x_i) + g(x_i)u_i \overset{\Delta}{=} F(x_i,u_i) \tag{1}$$

where $x_i = [x_{i,1}, x_{i,2}, \ldots, x_{i,h}]^{\mathrm{T}} \in \mathbb{R}^n$, $h = 1,2,\ldots,n$, and $u_i \in \mathbb{R}$. $f(x_i)$ and $g(x_i)$ denote system functions, and u_i represents the control input, $f(0) = 0$, and $f(\cdot) + g(\cdot)u_i$ is Lipschitz continuous on the compact set Ψ.

Adding uncertainty yields the following uncertain system

$$\dot{x}_i = f(x_i) + g(x_i)u_i + \varphi_i(t) \overset{\Delta}{=} F(x_i,u_i) + \varphi_i(t) \tag{2}$$

where $\varphi_i(t)$ is the uncertainty associated with time t, which is bounded by the function $\beta_1(t)$.

For consensus control, the admissible controller u_i allows $\lim_{t\to\infty} x_i \to x_0$, and x_0 shows the state of leader generated by Ref. [16]

$$\dot{x}_0 = f(x_0) \tag{3}$$

In light of the neighbors and leader, the local consensus errors for Eqs. (1) and (2) can be expressed as ϑ_i and δ_i, respectively.

Taking the derivatives of ϑ_i and δ_i, we have

$$\begin{aligned}\dot{\vartheta}_i(x_i,u_i) = {} & (l_{ii}+b_i)(f(x_i,u_i) + g(x_i)u_i) - \\ & b_i f(x_0) - \sum_{j\in\mathcal{N}_i} a_{ij}\left(f\left(x_j,u_j\right) + g\left(x_j\right)u_j\right)\end{aligned} \tag{4}$$

$$\begin{aligned}&\dot{\delta}_i(x_i,u_i,\varphi_i(t)) = \\ &\quad (l_{ii}+b_i)(f(x_i,u_i) + g(x_i)u_i + \varphi_i(t)) - b_i f(x_0) - \\ &\quad \sum_{j\in\mathcal{N}_i} a_{ij}\left(f\left(x_j,u_j\right) + g\left(x_j\right)u_j + \varphi_j(t)\right)\end{aligned} \tag{5}$$

According to the L_2 gain condition, robust optimal consensus control can be achieved by designing u_i, such that the system state of Eq. (5) is L_2-bounded, thus yielding [26]

$$\int_0^\infty U_i(\delta_i,u_i)\,\mathrm{d}t \leqslant \sigma^2 \int_0^\infty \|\varphi_i(t)\|^2 \mathrm{d}t \tag{6}$$

where $U_i(\delta_i,u_i)$ is the utility function and $\sigma > 0$ is a constant.

Therefore, the performance index function of agent i can be further defined as

$$J_i(\delta_i(0), u_i, \varphi_i) = \int_0^\infty \left(U_i(\delta_i, u_i) - \sigma^2 \|\varphi_i(t)\|^2\right) \mathrm{d}t \tag{7}$$

where $\varphi_i(t)$ is abbreviated as φ_i, and specify $U_i(\delta_i, u_i, \varphi_i) = U_i(\delta_i, u_i) - \sigma^2 \|\varphi_i\|^2$.

Similarly, the performance index corresponding to Eq. (4) is as follows

$$J_i(\vartheta_i(0), u_i) = \int_0^\infty U_i(\vartheta_i, u_i)\, \mathrm{d}t \tag{8}$$

where $U_i(\vartheta_i, u_i)$ denotes the utility function of Eq. (4).

Remark 1 The proposed control protocol u_i should enable Eq. (1) to achieve optimal consensus control if $\varphi_i(t) = 0$, and it not only makes the agents attain a consensus but also ensures that Eq. (6) is satisfied when $\varphi_i(t) \neq 0$.

III. Implementation of Event-Based Robust Optimal Consensus Control

To implement parallel control, the virtual control law is introduced to construct the feedback system in this section, and the event-triggering mechanism is presented to optimize the occupation of communication resources.

A. Application of Parallel Control

For parallel control, as detailed in Refs. [27, 28], a control system $\dot{u}_i$ that parallels the physical system should be developed. For this reason, based on the integral backstepping technique, a virtual controller w_i is built for Eq. (2)

$$w_i = \phi \dot{u}_i(x_i, u_i) - \theta_i(t) \tag{9}$$

where ϕ represents a constant and $\theta_i(t)$ is an auxiliary variable. Accordingly, the feedback system of Eq. (2) can be constructed as $\dot{u}_i(x_i, u_i) = 1/\phi(w_i + \theta_i(t))$.

Furthermore, $w_i = \phi \dot{u}_i(x_i, u_i)$ can be designed to construct the following control system for the nominal Eq. (1)

$$\dot{u}_i(x_i, u_i) = 1/\phi w_i \tag{10}$$

Considering Eqs. (4) and (5) and defining the augmented states $\bar{\vartheta}_i = \left[\vartheta_i^{\mathrm{T}}, u_i^{\mathrm{T}}\right]^{\mathrm{T}}$ and $\bar{\delta}_i = \left[\delta_i^{\mathrm{T}}, u_i^{\mathrm{T}}\right]^{\mathrm{T}}$, we have

$$\dot{\bar{\vartheta}}_i = \begin{bmatrix} \dot{\vartheta}_i(\vartheta_i, u_i) \\ 0 \end{bmatrix} + \begin{bmatrix} 0_n \\ 1/\phi \end{bmatrix} w_i = F\left(\bar{\vartheta}_i\right) + G\left(\bar{\vartheta}_i\right) w_i \tag{11}$$

$$\dot{\bar{\delta}}_i = \begin{bmatrix} \dot{\delta}_i(\delta_i, u_i, \varphi_i) \\ \theta_i(t) \end{bmatrix} + \begin{bmatrix} 0_n \\ 1/\phi \end{bmatrix} w_i = F\left(\bar{\delta}_i\right) + G\left(\bar{\delta}_i\right) w_i \tag{12}$$

where $G\left(\bar{\delta}_i\right) = G\left(\bar{\vartheta}_i\right) = \begin{bmatrix} 0_n, 1/\phi \end{bmatrix}^{\mathrm{T}}$.

Therefore, the optimal consensus control aims to design w_i and $\theta_i(t)$, so that agents synchronize with the leader while minimizing the augmented performance index as

$$\begin{aligned} \mathcal{J}_i(\vartheta_i(0), u_i, w_i) = \int_0^\infty \bar{U}_i\left(\bar{\vartheta}_i, w_i\right) \mathrm{d}t = \\ \int_0^\infty \left(U_i(\vartheta_i, u_i) + w_i^{\mathrm{T}} P w_i\right) \mathrm{d}t \end{aligned} \tag{13}$$

$$\begin{aligned} \mathcal{J}_i(\delta_i(0), u_i, \varphi_i, w_i) = \int_0^\infty \bar{U}_i\left(\bar{\delta}_i, \varphi_i, w_i\right) \mathrm{d}t = \\ \int_0^\infty \left(U_i(\delta_i, u_i, \varphi_i) + w_i^{\mathrm{T}} P w_i\right) \mathrm{d}t \end{aligned} \tag{14}$$

where $P = \rho^{\mathrm{T}} \rho$ represents a symmetric positive definite matrix formed by the invertible matrix ρ, and

$$U_i(\vartheta_i, u_i) = \bar{\vartheta}_i^{\mathrm{T}} Q_{ii} \bar{\vartheta}_i = \vartheta_i^{\mathrm{T}} Q \vartheta_i + u_i^{\mathrm{T}} R u_i,$$

where $Q > 0$ and $R > 0$ are the weight matrices, and $Q_{ii} = \mathrm{diag}\{Q, R\} \in \mathbb{R}^{(n+1)\times(n+1)}$.

Remark 2 It has been proved in Refs. [27, 28] that the asymptotic stability of Eqs. (11) and (12) can be guaranteed if and only if Eqs. (4) and (5) are asymptotically stable.

It follows from Eqs. (13) and (14) that the value functions can be identified as

$$V_i\left(\bar{\vartheta}_i(0)\right) = \int_0^\infty \bar{U}_i\left(\bar{\vartheta}_i, w_i\right) \mathrm{d}t \tag{15}$$

$$V_i\left(\bar{\delta}_i(0)\right) = \int_0^\infty \bar{U}_i\left(\bar{\delta}_i, \varphi_i, w_i\right) \mathrm{d}t \tag{16}$$

The admissible control rule w_i allows the optimal value functions to be defined with the optimal control rule w_i^* as

$$V_i^*\left(\bar{\vartheta}_i(0)\right) = \min_{w_i} \mathcal{J}_i\left(\bar{\vartheta}_i(0), w_i\right) = \int_0^\infty \bar{U}_i\left(\bar{\vartheta}_i, w_i^*\right) \mathrm{d}t \tag{17}$$

$$V_i^*\left(\bar{\delta}_i(0)\right) = \min_{w_i} \mathcal{J}_i\left(\bar{\delta}_i(0), \varphi_i, w_i\right) = \int_0^\infty \bar{U}_i\left(\bar{\delta}_i, \varphi_i, w_i^*\right) \mathrm{d}t \tag{18}$$

Taking the partial derivatives of $V_i\left(\bar{\vartheta}_i\right)$ and $V_i\left(\bar{\delta}_i\right)$ with respect to $\bar{\vartheta}_i$ and $\bar{\delta}_i$, respectively, $V_{\bar{\vartheta}_i} = \left[V_{\vartheta_i}^{\mathrm{T}}, V_{u_i}^{\mathrm{T}}\right]^{\mathrm{T}}$ and $V_{\bar{\delta}_i} = \left[V_{\delta_i}^{\mathrm{T}}, V_{u_i}^{\mathrm{T}}\right]^{\mathrm{T}}$ can be obtained to construct the coupled Hamiltonian for Eqs. (11) and (12) as

$$\begin{aligned} & H_i\left(\bar{\vartheta}_i, V_{\bar{\vartheta}_i}, w_i\right) = \\ & V_{\vartheta_i}^{\mathrm{T}}\Big((l_{ii} + b_i)(f(x_i) + g_i(x_i) u_i) - b_i f(x_0) - \\ & \sum_{j \in \mathcal{N}_i} a_{ij}\left(f\left(x_j\right) + g_j\left(x_j\right) u_j\right)\Big) + \\ & 1/\phi V_{u_i}^{\mathrm{T}} w_i + U_i(\vartheta_i, u_i) + w_i^{\mathrm{T}} P w_i \end{aligned} \tag{19}$$

$$\begin{aligned} & H_i\left(\bar{\delta}_i, V_{\bar{\delta}_i}, w_i, \varphi_i(t)\right) = \\ & V_{\delta_i}^{\mathrm{T}}\Big((l_{ii} + b_i)(f(x_i) + g_i(x_i) u_i + \varphi_i(t)) - b_i f(x_0) - \\ & \sum_{j \in \mathcal{N}_i} a_{ij}\left(f\left(x_j\right) + g_j\left(x_j\right) u_j + \varphi_j(t)\right)\Big) + \\ & 1/\phi V_{u_i}^{\mathrm{T}}(w_i + \theta_i(t)) + U_i(\delta_i, u_i, \varphi_i(t)) + w_i^{\mathrm{T}} P w_i \end{aligned} \tag{20}$$

In light of the Bellman optimality principle, we have

$$H_i\left(\bar{\vartheta}_i, V_{\bar{\vartheta}_i}^*, w_i^*\right) = 0 \tag{21}$$

$$H_i\left(\bar{\delta}_i, V_{\bar{\delta}_i}^*, w_i^*, \varphi_i\right) = 0 \tag{22}$$

Then, it can be deduced from Eqs. (19)–(22) that $H_i\left(\bar{\delta}_i, V_{\bar{\delta}_i}^*, w_i^*, \varphi_i\right) = H_i\left(\bar{\vartheta}_i, V_{\bar{\vartheta}_i}^*, w_i^*\right)$ holds if there exists

$$V_{\delta_i}^{*\mathrm{T}}\left((l_{ii}+b_i)\varphi_i(t)-\sum_{j\in\mathcal{N}_i}a_{ij}\varphi_j(t)\right)+\frac{V_{u_i}^{*\mathrm{T}}\theta_i(t)}{\phi}=0 \tag{23}$$

Moreover, the auxiliary variable can be formed as

$$\theta_i(t)=-\phi\left(V_{u_i}^{*\mathrm{T}}V_{u_i}^{*}\right)^{-1}\left(V_{\delta_i}^{*\mathrm{T}}V_{u_i}^{*}\right)\left((l_{ii}+b_i)\varphi_i(t)-\sum_{j\in\mathcal{N}_i}a_{ij}\varphi_j(t)\right) \tag{24}$$

According to $\partial H_i\left(\bar{\vartheta}_i,V_{\bar{\vartheta}_i}^{*},w_i\right)/\partial w_i=0$, the optimal value function in Eq. (17) gives the following optimal control rule

$$w_i^{*}=-\frac{1}{2}P^{-1}G\left(\bar{\vartheta}_i\right)^{\mathrm{T}}V_{\bar{\vartheta}_i}^{*} \tag{25}$$

The equivalence relationship between Eqs. (21) and (22) implies that the feedback system in Eq. (5) can be formed by substituting Eqs. (24) and (25) into $\dot{u}_i^{*}(x_i,u_i)=1/\phi(w_i^{*}+\theta_i(t))$, achieving the optimal consensus control of uncertain Eq. (2).

B. Event-Based Online Implementation

This section attempts to perform optimal consensus control online based on a critic NN. It is worth noting that updating the NN weights in light of events enables the proposed method to be more efficient in terms of communication resources.

The critic NN can be expressed as

$$V_i^{*}\left(\bar{\vartheta}_i\right)=\omega_{\mathrm{c},i}^{*\mathrm{T}}\sigma_{\mathrm{c}}\left(\bar{\vartheta}_i\right)+\varepsilon_{\mathrm{c}}\left(\bar{\vartheta}_i\right) \tag{26}$$

where $\omega_{\mathrm{c},i}^{*}\in\mathbb{R}^{l\times 1}$ denotes the ideal weight, $\sigma_{\mathrm{c}}\left(\bar{\vartheta}_i\right)$ and ε_{c} are the activation function and approximate error, respectively.

The optimal value function generated from the output of critic NNs is

$$\hat{V}_i\left(\bar{\vartheta}_i\right)=\hat{\omega}_{\mathrm{c},i}^{\mathrm{T}}\sigma_{\mathrm{c}}\left(\bar{\vartheta}_i\right) \tag{27}$$

For event-based control, the monotonically increasing triggering instant sequence of agent i $\left\{t_k^i\right\}_{k=0}^{\infty}$, $t_0^i=0$ should be determined. Therefore, the sampled state $\hat{\bar{\vartheta}}_{i,k}\triangleq\bar{\vartheta}_i\left(t_k^i\right)$ is specified to identify the event-triggered error $e_k^i\triangleq e\left(t_k^i\right)$ as

$$e_k^i(t)=\hat{\bar{\vartheta}}_{i,k}-\bar{\vartheta}_i(t),\ t\in\left[t_k^i,t_{k+1}^i\right) \tag{28}$$

The sampled augmented consensus error system for Eq. (11) is provided as

$$\dot{\bar{\vartheta}}_i(t)=F_i\left(\bar{\vartheta}_i\right)+G\left(\bar{\vartheta}_i\right)w_i\left(e_k^i(t)+\bar{\vartheta}_i(t)\right) \tag{29}$$

Remark 3 The controller w_i is updated and the triggering error is reset to 0 only at the triggering instant. In the implementation, the zero-order hold (ZOH) maintains the updated w_i as a constant in $\left[t_k^i,t_{k+1}^i\right)$, i.e., $w_i(t)=w_i\left(\hat{\bar{\vartheta}}_{i,k}\right)$, $\forall t\in\left[t_k^i,t_{k+1}^i\right)$.

Taking the gradient of $V_i^{*}\left(\bar{\vartheta}_i\right)$, i.e., $V_{\bar{\vartheta}_i}^{*}$, we have

$$w_{i,k}^{*}\triangleq w_i^{*}\left(\hat{\bar{\vartheta}}_{i,k}\right)=-\frac{1}{2}P^{-1}G^{\mathrm{T}}\left(\hat{\bar{\vartheta}}_{i,k}\right)V_{\bar{\vartheta}_i}^{*}\left(t_k^i\right) \tag{30}$$

Referring to Eq. (27), Eq. (30) is reformulated as

$$\hat{w}_i\left(\hat{\bar{\vartheta}}_{i,k}\right)=-\frac{1}{2}P^{-1}G^{\mathrm{T}}\left(\hat{\bar{\vartheta}}_{i,k}\right)\left(\nabla\sigma_{\mathrm{c}}\left(\hat{\bar{\vartheta}}_{i,k}\right)\right)^{\mathrm{T}}\hat{\omega}_{\mathrm{c},i} \tag{31}$$

To tune the NN weights, the residual error function $E_{\mathrm{c},i}=e_{\mathrm{c},i}^2/2$ is constructed with

$$\begin{aligned}e_{\mathrm{c},i}\triangleq{}&\hat{H}_i\left(\bar{\vartheta}_i,\hat{w}_i\left(\hat{\bar{\vartheta}}_{i,k}\right),\hat{\omega}_{\mathrm{c},i}\right)=\\&\bar{\vartheta}_i^{\mathrm{T}}Q_{ii}\bar{\vartheta}_i+\hat{w}_i^{\mathrm{T}}\left(\hat{\bar{\vartheta}}_{i,k}\right)P\hat{w}_i\left(\hat{\bar{\vartheta}}_{i,k}\right)+\\&\hat{\omega}_{\mathrm{c},i}^{\mathrm{T}}\left(\nabla\sigma_{\mathrm{c}}\left(\bar{\vartheta}_i\right)\right)\left(F\left(\bar{\vartheta}_i\right)+G\left(\bar{\vartheta}_i\right)\hat{w}_i\left(\hat{\bar{\vartheta}}_{i,k}\right)\right)\end{aligned} \tag{32}$$

Then, the gradient descent scheme yields the update law as

$$\begin{aligned}\dot{\hat{\omega}}_{\mathrm{c},i}={}&-\alpha_{\mathrm{c},i}\frac{1}{(1+\gamma^{\mathrm{T}}\gamma)^2}\left(\frac{\partial E_{\mathrm{c},i}}{\partial\hat{\omega}_{\mathrm{c},i}}\right)=\\&-\frac{\alpha_{\mathrm{c},i}\gamma}{(1+\gamma^{\mathrm{T}}\gamma)^2}\left(\bar{\vartheta}_i^{\mathrm{T}}Q_{ii}\bar{\vartheta}_i+\hat{w}_i^{\mathrm{T}}\left(\hat{\bar{\vartheta}}_{i,k}\right)P\hat{w}_i\left(\hat{\bar{\vartheta}}_{i,k}\right)+\gamma^{\mathrm{T}}\hat{\omega}_{\mathrm{c},i}\right)\end{aligned} \tag{33}$$

where $\gamma=\nabla\sigma_{\mathrm{c}}\left(\bar{\vartheta}_i\right)\left(F\left(\bar{\vartheta}_i\right)+G\left(\bar{\vartheta}_i\right)\hat{w}_i\left(\hat{\bar{\vartheta}}_{i,k}\right)\right)\in\mathbb{R}^{l}$, and $\alpha_{\mathrm{c},i}>0$ is a constant.

Define the weight estimation error as $\tilde{\omega}_{\mathrm{c},i}=\omega_{\mathrm{c},i}^{*}-\hat{\omega}_{\mathrm{c},i}$, and its dynamics can be calculated as

$$\begin{aligned}\dot{\tilde{\omega}}_{\mathrm{c},i}={}&-\dot{\hat{\omega}}_{\mathrm{c},i}=\\&-\frac{\gamma^2\alpha_{\mathrm{c},i}\omega_{\mathrm{c},i}^{*\mathrm{T}}}{(1+\gamma^{\mathrm{T}}\gamma)^2}+\frac{\gamma^2\alpha_{\mathrm{c},i}\hat{\omega}_{\mathrm{c},i}^{\mathrm{T}}}{(1+\gamma^{\mathrm{T}}\gamma)^2}+\\&\frac{\gamma\alpha_{\mathrm{c},i}\nabla\varepsilon_{\mathrm{c}}\left(\bar{\vartheta}_i\right)}{(1+\gamma^{\mathrm{T}}\gamma)^2}\left(F\left(\bar{\vartheta}_i\right)+G\left(\bar{\vartheta}_i\right)\hat{w}_i\left(\hat{\bar{\vartheta}}_{i,k}\right)\right)=\\&-\alpha_{\mathrm{c},i}\frac{\gamma}{(1+\gamma^{\mathrm{T}}\gamma)^2}\left(\gamma^{\mathrm{T}}\tilde{\omega}_{\mathrm{c},i}-e_{\mathrm{cH}}\right)\end{aligned} \tag{34}$$

where $e_{\mathrm{cH}}=-\nabla\varepsilon_{\mathrm{c}}\left(\bar{\vartheta}_i\right)\left(F\left(\bar{\vartheta}_i\right)+G\left(\bar{\vartheta}_i\right)\hat{w}_i\left(\hat{\bar{\vartheta}}_{i,k}\right)\right)$.

The optimal control of augmented Eq. (11) achieved by Eq. (31) indicates that the control input deduced from Eq. (31) is capable of performing optimal consensus control of Eq. (1). According to the integral technique, the optimal consensus controller of uncertain Eq. (2) can also be generated from the feedback system composed of Eqs. (24) and (31) as

$$u_i=u_i(0)+1/\phi\int_0^t\left(\hat{w}_i\left(\hat{\bar{\vartheta}}_{i,k}\right)+\theta_i(t)\right)\mathrm{d}\tau \tag{35}$$

where $\tau\in[0,t]$, and time-triggered $\varphi_i(t)$ gives the time-triggered $\theta_i(t)$.

Then, the triggering instants are determined by developing the triggering condition based on the system stability.

Referring to Refs. [29, 30], a non-negative function η_x is utilized to build a dynamic event-triggered mechanism that increases the triggering interval, whose dynamics is as follows

$$\begin{aligned}\dot{\eta}_x={}&-\lambda\eta_x-2\lambda_1\|\rho\|^2\left\|P^{-1}\right\|^2\left\|\hat{\omega}_{\mathrm{c},i}\right\|^2\left\|e_k^i\right\|^2+\\&\left(1-\beta_2^2\right)\lambda_{\min}(Q_{ii})\left\|\bar{\vartheta}_i\right\|^2\end{aligned} \tag{36}$$

where λ and λ_1 denote positive constants, $\beta_2\in(0,1)$, and $\eta_x(0)\geqslant 0$.

Assumption 1 (1) The gradients of the activation function and estimation error are bounded by $\left\|\nabla\sigma_{\mathrm{c}}\left(\bar{\vartheta}_i\right)\right\|\leqslant\lambda_\sigma$ and $\left\|\nabla\varepsilon_{\mathrm{c}}\left(\bar{\vartheta}_i\right)\right\|\leqslant\lambda_\varepsilon$, where $\lambda_\sigma>0$ and $\lambda_\varepsilon>0$. In addition, $|e_{\mathrm{cH}}|\leqslant\lambda_e$, $\lambda_e>0$ holds, there exists

$$\left\|\nabla\sigma_{\mathrm{c}}\left(\bar{\vartheta}_i(t)\right)-\nabla\sigma_{\mathrm{c}}\left(\hat{\bar{\vartheta}}_{i,k}\right)\right\|\leqslant L_2\left\|e_k^i\right\| \tag{37}$$

where $L_2>0$ is a constant.

(2) The following Lipschitz continuity is satisfied

$$\left\|G\left(\bar{\vartheta}_i\right)-G\left(\hat{\bar{\vartheta}}_{i,k}\right)\right\| \leqslant L_3\left\|e_k^i\right\| \tag{38}$$

where $\left\|G\left(\bar{\vartheta}_i\right)\right\| \leqslant \lambda_g$, and L_3 and λ_g are positive constants.

Consequently, it can be concluded that Eq. (11) forced by the event-based control rule Eq. (31) with the tuning law Eq. (33) is asymptotically stable, which also allows the weight estimation error of critic NN $\tilde{\omega}_{\mathrm{c},i}$ to be uniformly ultimately bounded (UUB) if there exist

$$\left\|\tilde{\omega}_{\mathrm{c},i}\right\| \geqslant \sqrt{\lambda_3/\lambda_2} \tag{39}$$

$$\left\|e_k^i\right\|^2 \leqslant \frac{\left(1-\beta_2^2\right)\lambda_{\min}\left(Q_{ii}\right)\left\|\bar{\vartheta}_i\right\|^2}{2\lambda_1\|\rho\|^2\left\|P^{-1}\right\|^2\left\|\hat{\omega}_{\mathrm{c},i}\right\|^2}+ \frac{\eta_x}{2\mu\lambda_1\|\rho\|^2\left\|P^{-1}\right\|^2\left\|\hat{\omega}_{\mathrm{c},i}\right\|^2} \triangleq \hat{e}_{\mathrm{T}} \tag{40}$$

where $\lambda_1=L_3^2\lambda_\sigma^2+L_2^2\lambda_g^2$, $\lambda_3=2\|\rho\|^2\left\|P^{-1}\right\|^2\lambda_g^2\lambda_\varepsilon^2+\alpha_{\mathrm{c},i}^2\lambda_e^2/2$, $\lambda_2=\left(\alpha_{\mathrm{c},i}-1/2\right)\lambda_{\min}\left(\gamma_1^{\mathrm{T}}\gamma_1\right)-2\|\rho\|^2\left\|P^{-1}\right\|^2\lambda_g^2\lambda_\sigma^2$, $\gamma_1=\gamma/\left(1+\gamma^{\mathrm{T}}\gamma\right)$, and μ is a designable parameter.

Referring to Ref. [15], the local Lyapunov function can be selected as

$$L_i=L_{i1}+L_{i2}+L_{i3}+L_{i4} \tag{41}$$

where $L_{i1}=V_i^*\left(\bar{\vartheta}_i\right)$, $L_{i2}=V_i^*\left(\hat{\bar{\vartheta}}_{i,k}\right)$, $L_{i3}=\tilde{\omega}_{\mathrm{c},i}^{\mathrm{T}}\tilde{\omega}_{\mathrm{c},i}/2$, and $L_{i4}=\eta_x$.

The analysis is usually carried out in the following two cases:

(1) Events are not triggered, $\forall t\in\left(t_k^i,t_{k+1}^i\right)$ and $\dot{L}_{i2}=0$.

$$\begin{cases} \dot{L}_{i1}=\left(\nabla V_i^*\left(\bar{\vartheta}_i\right)\right)^{\mathrm{T}}\left(F\left(\bar{\vartheta}_i\right)+G\left(\bar{\vartheta}_i\right)w_i\left(\hat{\bar{\vartheta}}_{i,k}\right)\right), \\ \dot{L}_{i3}=-\alpha_{\mathrm{c},i}\tilde{\omega}_{\mathrm{c},i}^{\mathrm{T}}\dfrac{\gamma}{\left(1+\gamma^{\mathrm{T}}\gamma\right)^2}\left(\phi^{\mathrm{T}}\tilde{\omega}_{\mathrm{c},i}-e_{\mathrm{cH}}\right) \end{cases} \tag{42}$$

It follows from Eqs. (25)–(27), (30), and (31) that

$$\dot{L}_{i1}\leqslant-\bar{\vartheta}_i^{\mathrm{T}}Q_{ii}\bar{\vartheta}_i-\left\|\rho\hat{w}_i\left(\hat{\bar{\vartheta}}_{i,k}\right)\right\|^2+ 2\|\rho\|^2\left\|P^{-1}\right\|^2\left(\lambda_1\left\|e_k^i\right\|^2\left\|\hat{\omega}_{\mathrm{c},i}\right\|^2+\lambda_g^2\left(\lambda_\sigma^2\left\|\tilde{\omega}_{\mathrm{c},i}\right\|^2+\lambda_\varepsilon^2\right)\right) \tag{43}$$

Taking the derivative of L_{i3}, we have

$$\dot{L}_{i3}\leqslant-\left(\alpha_{\mathrm{c},i}-\frac{1}{2}\right)\lambda_{\min}\left(\gamma_1^{\mathrm{T}}\gamma_1\right)\left\|\tilde{\omega}_{\mathrm{c},i}\right\|^2+\frac{1}{2}\alpha_{\mathrm{c},i}^2\lambda_e^2 \tag{44}$$

It can be deduced from Eqs. (35), (44), and (45) that

$$\dot{L}_i\leqslant-\lambda_{\min}\left(Q_{ii}\right)\left\|\bar{\vartheta}_i\right\|^2+\left(\beta_2^2-1\right)\lambda_{\min}\left(Q_{ii}\right)\left\|\bar{\vartheta}_i\right\|^2- \lambda\eta_x-\lambda_2\left\|\tilde{\omega}_{\mathrm{c},i}\right\|^2+\lambda_3+2\lambda_1\|\rho\|^2\left\|P^{-1}\right\|^2\left\|\hat{\omega}_{\mathrm{c},i}\right\|^2\left\|e_k^i\right\|^2 \tag{45}$$

(2) An event is triggered, i.e., $\forall t=t_{k+1}^i$.

In this case, the Lyapunov function is formed as

$$\Delta L_i=L_2\left(\hat{\bar{\vartheta}}_{i,k+1}\right)-L_2\left(\bar{\vartheta}_i\left(t_{k+1}^-\right)\right)=\Delta L_{i1}+\Delta L_{i2}+\Delta L_{i3}+\Delta L_{i4} \tag{46}$$

where $\bar{\vartheta}_i\left(t_{k+1}^-\right)=\lim\limits_{\tau\to0}\bar{\vartheta}_i\left(t_{k+1}-\tau\right)$, $t_{(k+1)}^-$ indicates that the time tends to $t_{(k+1)}^-$ from negative direction, i.e., $t\in\left(t_k^i,t_{k+1}^i\right)$.

Since Eqs. (39) and (40) are satisfied, $\dot{L}_i<0$ indicates that the Lyapunov function is monotonically decreasing.

Thus, $\Delta L_{i2}\leqslant-K\left(\left\|\hat{\bar{\vartheta}}_{i,k+1}-\hat{\bar{\vartheta}}_{i,k}\right\|\right)$ can be deduced from $\Delta L_{i2}=V_i^*\left(\hat{\bar{\vartheta}}_{i,k+1}\right)-V_i^*\left(\hat{\bar{\vartheta}}_{i,k}\right)$, where $K(\cdot)$ is a class-$\mathcal{K}$ function. Furthermore, $\Delta L_i<0$ can be obtained to guarantee the stability of Eq. (11).

IV. Simulation

This section provides a numerical example to verify the validity of the theoretical results.

The MAS with three agents is employed as shown in Fig. 1, and there is information transmitted between the leader and agent 1.

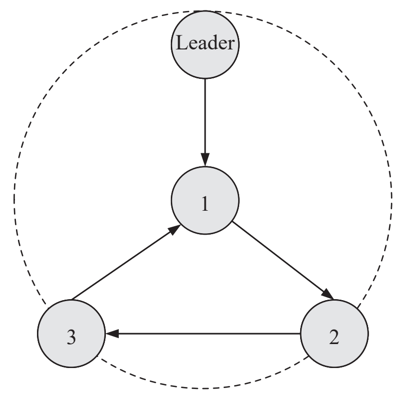

Figure 1 Communication diagraph.

The uncertain system in Eq. (2) is provided as

$$\begin{cases} \dot{x}_{i,1}=-x_{i,1}+x_{i,2}+2x_{i,2}^3, \\ \dot{x}_{i,2}=-0.5\left(x_{i,1}+x_{i,2}\right)+0.5x_{i,2}\left(1+2x_{i,2}^2\right)\sin^2\left(x_{i,1}\right)+ \\ \qquad \sin\left(x_{i,1}\right)u_i+\varphi_i(t) \end{cases} \tag{47}$$

and the trajectory of the leader is generated from

$$\begin{cases} \dot{x}_{0,1}=-x_{0,1}+x_{0,2}+2x_{0,2}^3, \\ \dot{x}_{0,2}=-0.5\left(x_{0,1}+x_{0,2}\right)+0.5x_{0,2}\left(1+2x_{0,2}^2\right)\sin^2\left(x_{0,1}\right) \end{cases} \tag{48}$$

where $x_i=\left[x_{i,1},x_{i,2}\right]^{\mathrm{T}}\in\mathbb{R}^2$, $u_i\in\mathbb{R}$, $i=1,2,3$, and $\varphi_i(t)=0.5p_1x_{i,1}\sin\left(p_2x_{i,2}\right)$, where p_1 and $p_2\in[-1,1]$ are unknown parameters. The boundary function is $\beta_1(t)=\|x_i\|$. Let $x_{i,0}=\left[x_{1,0}^{\mathrm{T}},x_{2,0}^{\mathrm{T}},x_{3,0}^{\mathrm{T}}\right]^{\mathrm{T}}=[-0.75,0.30,0.50,0.50,-0.25,0.50]^{\mathrm{T}}$ and the initial state of leader be $[0.25,0.10]^{\mathrm{T}}$.

Moreover, $\phi=1$, $\alpha_{\mathrm{c},i}=10$, $Q=I_2$, $R=2I$, and $P=I$ are selected, where I_n represents the n-dimensional identity matrix. $\sigma_{\mathrm{c}}\left(\bar{\vartheta}_i\right)=\left[\vartheta_{i1}^2,\vartheta_{i1}\vartheta_{i2},\vartheta_{i1}u_i,\vartheta_{i2}^2,\vartheta_{i2}u_i,u_i^2\right]^{\mathrm{T}}$, $l=6$, and the initial weights are taken as $\omega_{\mathrm{c},1}=0.10\times1_6$, $\omega_{\mathrm{c},2}=0.15\times1_6$, and $\omega_{\mathrm{c},3}=0.20\times1_6$. Over time, the weights converge to $\hat{\omega}_{\mathrm{c},1}=[0.1598,0.1478,0.1495,0.1510,0.1502,0.1500]^{\mathrm{T}}$, $\hat{\omega}_{\mathrm{c},2}=[0.1807,0.1713,0.1702,0.1702,0.1701,0.1700]^{\mathrm{T}}$, and $\hat{\omega}_{\mathrm{c},3}=[0.1984,0.1904,0.1895,0.1900,0.1900,0.1900]^{\mathrm{T}}$.

As depicted in Fig. 2, agents in MAS have attained a consensus following the leader, where different colors identify agents. Figure 3 draws the gap $\left\|e_k^i\right\|^2$ and the threshold function. It is clear that the gap is always smaller than the threshold value, indicating that Eq. (40) is satisfied and the stability of the system is maintained. Thus, the effectiveness of the event-based robust optimal consensus control developed in this study is verified.

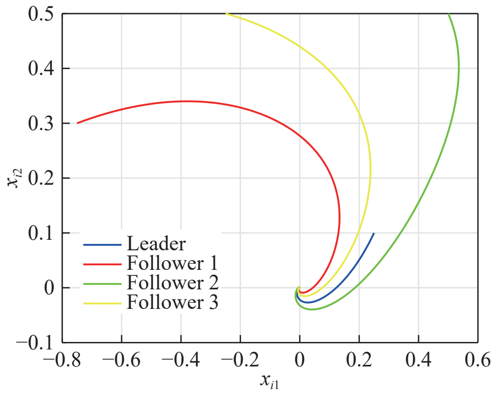

Figure 2 State trajectory (p_1 = 1.0 and p_2 = 0.5).

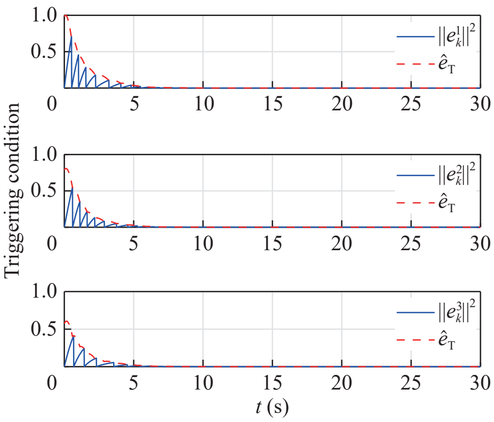

Figure 3 Evolution of triggering condition.

V. Conclusion

This investigation identified a robust optimal consensus control strategy via parallel control. Referring to the optimality principle, the relationship between the optimal control for the nominal and uncertain systems delivers an auxiliary variable, which is capable of reducing the negative effect of uncertainties on the system. The integral backstepping technique motivates the construction of a virtual control law, which is estimated by a critic NN tuned in response to events. Then, the robust optimal consensus control is implemented online using the feedback system built on the virtual control law and auxiliary variable. Furthermore, the event-triggered mechanism with a non-negative function presents fewer triggers. Finally, simulation findings confirm the contributions. Further research is required to determine more intelligent event-triggered mechanisms, thus facilitating the advancement of event-based control.

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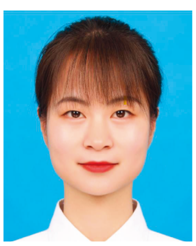

Shanshan Jiao received the MS degree in control theory and control engineering from Xi'an University of Technology, China, in 2021. She is currently pursuing the PhD degree in intelligent science and systems at Institute of Systems Engineering, Macau University of Science and Technology, Macao, China. Her research interests include adaptive dynamic programming and neural network based control.

Qinglai Wei received the BS degree in automation and the PhD degree in control theory and control engineering from Northeastern University, Shenyang, China, in 2002 and 2009, respectively. He is currently a professor at State Key Laboratory of Multimodal Artificial Intelligence Systems, Institute of Automation, Chinese Academy of Sciences, Beijing, China. His research interests include adaptive dynamic programming, neural network based control, optimal control, nonlinear systems, and their industrial applications.

Fei Dai received the BS degree in automation from Wuhan University of Technology, China, in 2005. He is currently a director at Guangzhou Goaland Energy Conservation Tech. Co., Ltd., China. His research interests include electrical equipment cooling technology, new product development, and management work.

Jianchao Wu received the BS degree in electrical engineering and automation from South China University of Technology, China, in 2016. He is currently a director at Guangzhou Goaland Energy Conservation Tech. Co., Ltd., China. His research interests include construction and implementation of the product quality and safety management system.

Miaosheng Qiu received the MS degree in energy and power engineering from Guangdong University of Technology, China, in 2017. He is currently a director at Guangzhou Goaland Energy Conservation Tech. Co., Ltd., China. His research interests include design of power electronic water cooling equipment.

Bin Zhang received the BS degree in electrical engineering and automation from South China University of Technology, China, in 2018. He is currently an electrical expert at Guangzhou Goaland Energy Conservation Tech. Co., Ltd., China. His research interests include electrical engineering research and standardization construction.

Yongjin Luo received the BS degree from China University of Mining and Technology, China, in 2012. He is currently an intermediate engineer at Guangzhou Goaland Energy Conservation Tech. Co., Ltd., China. His research interests include development of converter valve cooling systems for high-voltage direct current (HVDC) power transmission projects and design of liquid cooling systems for data centers.

Kunxin Huang received the MS degree in control theory and control engineering from Xiangtan University, China, in 2013. He holds an intermediate professional title as an electrical engineer and is an electrical expert at Guangzhou Goaland Energy Conservation Tech. Co., Ltd., China. His research interests include automation control, valve cooling systems, and the application of valve cooling control systems in HVDC power transmission projects.

Genpo Ma received the BS degree from North China University of Technology, China, in 1992. He is currently an intermediate engineer at Guangzhou Goaland Energy Conservation Tech. Co., Ltd., China. His research interests include development of converter valve cooling systems for HVDC transmission projects.