Data-Driven Switching Optimal Control Strategy for Aircraft Engine Acceleration Dynamics

Zelin Li, Yan Shi, Shuoshuo Liu, Tao Sun, Xudong Zhao, and Lijuan Liu

Abstract—**The transient-state acceleration control of aircraft engines has consistently been a focal point in the field of aircraft engine control research. To address the controller design problem of transient-state acceleration in aircraft engines based on switched linear systems, a data-driven optimization control algorithm is proposed. First, the optimization control problem for the transient-state acceleration process of aircraft engines is mathematically described. Subsequently, the Bellman optimal criterion in optimal control is utilized to derive the invariance of a value function at switching instants, providing an initial value for the optimization of the next subsystem. The optimization problem is then transformed into solving a Lyapunov equation using the principles of dynamic programming, and an iterative adaptive dynamic programming (ADP) algorithm is introduced to obtain optimal feedback control gains, which makes the solution of Lyapunov equation approach the optimal solution infinitely. Finally, simulation validation is conducted on a switched linear system for the transient-state acceleration of a specific aircraft engine. The effectiveness and superiority of the proposed algorithm are verified by comparing to traditional proportion integral differential (PID) control algorithms and active disturbance rejection control (ADRC).**

Index Terms—**Aircraft engine, transient-state, switched linear system, data-driven approach, adaptive dynamic programming, optimal control**

I. Introduction

With the continuous improvement of the performance requirements for aircraft engines, the design of aircraft engine control systems is also constantly evolving and improving. According to the changes in control setpoints, the control problems of aircraft engines can be roughly divided into two main categories: steady-state control and transient control. Steady-state control refers to the fact that when the external working conditions change, the control reference of the aircraft engine remains unchanged to ensure that the aircraft engine maintains a stable performance. On the contrary, transient control refers to the smooth, fast, and accurate transition of an aircraft engine from one initial state to another when the control reference is changed. In addition, transient control can be further subdivided into acceleration, deceleration, and thrust control problems, making the various transient performance characteristics become the key components of the overall performance of an aircraft engine. Taking the acceleration as an example, the acceleration capability of aircraft engine is a key metric for assessing aircraft engine performance, impacting aircraft takeoff, acceleration during flight, and maneuvering performance directly [1]. Especially for a military aircraft, operation conditions such as chasing enemy planes, evading adversaries, and climbing necessitate the use of the acceleration mode of aircraft engine. Due to the technical similarities between acceleration control and deceleration control of aircraft engines, and the fact that the acceleration process is subject to more constraints compared to deceleration, the control problem of the acceleration of aircraft engine has become one of the hot topics in the field of aircraft engine control research.

In fact, the study of aircraft engine acceleration process control holds significant practical importance. Particularly, the dynamic characteristics of aircraft engine will vary with changes of current operating conditions, essentially belonging to the domain of nonlinear control system problems. Currently, the predominant approach is utilizing nonlinear programming methods to achieve transient-state control of aircraft engines. However, in earlier stages, researchers focused on linearizing nonlinear models around a certain steady-state point in aircraft engine. For instance, employing the linear quadratic regulator (LQR) method, the jet aircraft model was treated as a linear multivariable system, and adaptive feedback control laws were employed to control the ground acceleration process of aircraft engine [2]. Due to the limited robustness of the linear quadratic regulator method, it may result in system instability under disturbance [3]. To address this issue, the linear quadratic Gaussian method was employed to achieve multivariable control for the GE-21 engine, enhancing its robustness [4]. In the actual aircraft engine system, due to its complex internal structure and variable working environment, the linear system cannot well characterize all dynamic properties of the aircraft engine. After years of development, more and more scholars have studied the transitional state control problem of aircraft engine as a nonlinear optimal control problem. For addressing optimization control [5–7] problems in nonlinear systems with constraints, the sequential quadratic programming algorithm is

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Zelin Li, Yan Shi, Shuoshuo Liu, and Xudong Zhao are with Key Laboratory of Intelligent Control and Optimization for Industrial Equipment (Dalian University of Technology), Ministry of Education, Dalian 116024, China (e-mail: bryanze@163.com; shiyan@dlut.edu.cn; dut_ls_2025@126.com; xdzhaohit@gmail.com).

Tao Sun is with School of Control Science and Engineering, Dalian University of Technology, Dalian 116024, China, and also with Department of Automation, Tsinghua University, Beijing 100084, China (e-mail: tao.sun.meng@gmail.com).

Lijuan Liu is with School of Software, Dalian Jiaotong University, Dalian 116028, China (e-mail: liulj@djtu.edu.cn).

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widely applied [8–10]. In the vicinity of the current iteration point, it constructs a quadratic programming subproblem by approximating the objective function and constraints quadratically. The solution to this subproblem provides a search direction for the next iteration point. However, the sequential quadratic programming algorithm belongs to the category of local optimization algorithms, which may fall into a local optimal solution. Additionally, it needs to solve quadratic programming subproblems at each iteration. This requirement leads to large-scale matrix computations. Consequently, it occupies more computational resources in practical applications. In the context of performance optimization of aircraft engines, the global optimal solution is particularly important for complex problems involving multiple variables and constraints [11, 12]. In addition, another algorithm widely used in aircraft engine transition state control is the model predictive control algorithm [13–17]. Its principle is to determine control decisions by predicting the future behavior of the system. A predictive model is used to forecast the response of the system over a future time horizon based on the current system state and control inputs. These predictions are utilized to formulate an optimization problem to determine the optimal control inputs. A key feature of model predictive control is that it can handle multivariable systems with constraints. However, model predictive control typically requires accurate mathematical models to establish the system dynamics. Any model errors can significantly impact control performance. For this reason, the model predictive control will suffer from severe performance degradation or infeasibility due to the high complexity of aircraft engine models and their susceptibility to external disturbances [18, 19]. Nowadays, with the rapid development of intelligent algorithms, researchers begin to apply them to optimization control. For instance, an online optimization controller for aircraft engines was proposed based on genetic algorithms, which was verified through joint simulation experiments to improve acceleration performance and thrust within the entire flight envelope [20]. In Ref. [21], a performance seeking control scheme was proposed based on the grey wolf optimization algorithm to improve the optimization accuracy. In Refs. [22, 23], better optimization results were produced by using particle swarm algorithm under optimization conditions at the maximum thrust. However, it relies on parameter adjustments and may not adapt well to multiple operating states. Although genetic algorithm, grey wolf algorithm, or particle swarm optimization algorithm are all intelligent algorithms of global optimization and can solve the problem that traditional sequential quadratic programming algorithms are prone to fall into local optimal solutions, they cannot solve the problem of occupying too much computing resources. Nevertheless, they may not effectively tackle issues related to excessive computational resource consumption. Besides, in Ref. [24], an optimization method was proposed based on soft actor-critic (SAC). However, SAC is primarily designed for standard reward structures and is not very efficient for complex, non-standard reward structures. By using the G-SAC intelligent optimization algorithm in the context of aero-engines, the effectiveness of low-pressure shaft speed control was verified [25]. However, the drawbacks of the G-SAC algorithm are apparent. For instance, the G-SAC algorithm requires training two neural networks, necessitating frequent gradient updates during the training process, which will lead to high computational complexity, especially in high-dimensional states. Additionally, the G-SAC method requires extensive adjustments and parameter tuning when dealing with systems with uncertain dynamics, such as aircraft engines.

Through the analysis above, it can be seen that there are following problems in the transient-state acceleration control of aircraft engine. How to tackle the problem that a single linear model cannot accurately characterize the dynamics with strong nonlinearity during the acceleration process? How to solve the problem of over-reliance on model accuracy in the controller design process? How to sort out the problem of excessive computation in the optimization process? At present, in the design of transient-state acceleration control for aircraft engines, the existing literature has hardly investigated the data-driven adaptive optimal acceleration control of linear switched models for aircraft engines [7, 26], which motivates us to explore further.

In this paper, appropriate steady state points are selected within the entire acceleration envelope of an aircraft engine and multiple steady state models are identified to describe the dynamic characteristics during the entire acceleration process. In addition, based on the advancement of artificial intelligence technology, the adaptive dynamic programming (ADP) algorithm is employed to design the controller from a data-driven perspective. One of the advantages of this method is that it does not depend on the model of the system in the process of controller design [27–31]. This advantage makes our controller design more flexible, because accurate system models are not easy to obtain and may change as working conditions change. This flexibility is more valuable in practical applications. In addition, this method has an obvious advantage that the optimal solution of the ADP method can be obtained by solving the Bellman equation or the Hamilton-Jacobi-Bellman equation, which has lower computational complexity and converges to the optimal solution faster [32–35]. Overall, this work helps to advance the research on aircraft engines acceleration control strategies by combining data-driven optimal controller design and the application of the ADP algorithm. The main contributions of this paper are as follows:

(1) By introducing the switched linear model of the aircraft engines and applying the optimal control theory, the value iteration recurrence relationship of the optimal control problem of the switched linear system is successfully derived.

(2) Based on the diversity of aircraft engines system operating data collected under proportion integral differential (PID) controller conditions and the time-varying disturbance signals acting on subsystems, a data-driven optimal control algorithm is proposed to achieve the goal of continuous optimization of feedback control gains through data learning and adjustments.

(3) On the aircraft engine switched linear model, the single PID controller, multi-switching PID controller, and active

disturbance rejection control (ADRC) are compared with the proposed optimal control strategy, which verifies that the proposed method has the advantages of faster response speed and zero overshoot.

The structure of this paper is outlined as follows. Section II introduces the transient-state acceleration process based on switched linear systems and its mathematical description. Section III proposes an iterative algorithm for switched linear systems based on data-driven approaches. Section IV presents the simulation validation of the proposed method on the transient-state acceleration process of a specific model of aircraft engine. Finally, Section V concludes the paper and provides prospects for subsequent research.

II. Transient-State Acceleration Process of Aircraft Engine

A. Introduction to the Transient-State Acceleration Process Based on Switched System

The transient-state operating mode of aircraft engines is a critical process for rapidly and precisely adjusting thrust to meet the requirements of flight. Generally, the transient-state mode mainly includes processes such as acceleration and deceleration, which means changing the state from one steady state to another steady state. To further illustrate the principles of transient-state control, Fig. 1 depicts a schematic of the compressor map of a certain aircraft engine during the transient-state process. In Fig. 1, points A, B, and C represent three steady-state points, corresponding to three steady-state systems. The red curve above the common operating line is the acceleration control curve, and the blue curve below is the deceleration control curve. Additionally, Fig. 1 contains a surge boundary line. When the aircraft engine needs to transition from state A to state C, it follows the acceleration curve. In this process, the initial state A is in the steady-state range of system A, so the aircraft engine operates in system A. When accelerating to point E, it enters the steady-state range of system B, triggering a switching to system B. Similarly, when accelerating to point F, it enters the steady-state range of system C, leading to a switching to system C. This process continues until reaching point C, completing the acceleration.

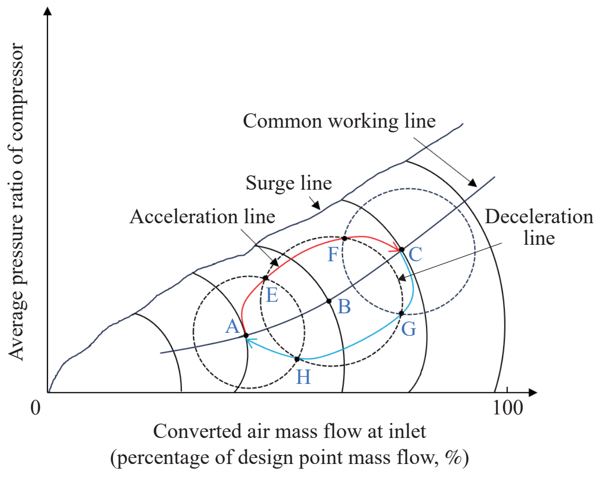

Figure 1 Transient-state process of the state on the compressor characteristic map.

B. Mathematical Description of Transient-State Acceleration Process Based on Switched System

The expression of the linear model of the aircraft engine transition state switching is shown as [36, 37]

$$\begin{bmatrix} \Delta\dot{n}_{\mathrm{h}} \\ \Delta\dot{n}_{\mathrm{l}} \end{bmatrix} = A_{\sigma(n_{\mathrm{h}}(t))}\begin{bmatrix} \Delta n_{\mathrm{h}} \\ \Delta n_{\mathrm{l}} \end{bmatrix} + B_{\sigma(n_{\mathrm{h}}(t))}\Delta q_{\mathrm{mf}} \tag{1}$$

where Δn_{h} and Δn_{l} represent the changes in the high and low pressure rotor speeds of the aircraft engine relative to the steady-state point, respectively, Δq_{mf} represents the amount of change in aircraft engine fuel quantity relative to the steady-state fuel quantity, which is the control input of the system. $A_{\sigma(n_{\mathrm{h}}(t))} \in \Re^{2\times 2}$ and $B_{\sigma(n_{\mathrm{h}}(t))} \in \Re^{2\times 1}$ represent the dynamic information of each subsystem. $\sigma(n_{\mathrm{h}}(t)) : [0,\infty) \to I = \{1,2,...,n\}$ is a switching signal that determines the activation of the i-th subsystem. For simplicity, Eq. (1) is written in

$$\dot{x}(t) = A_{\sigma(n_{\mathrm{h}}(t))}x(t) + B_{\sigma(n_{\mathrm{h}}(t))}u(t) \tag{2}$$

where

$$\sigma(n_{\mathrm{h}}(t)) = \begin{cases} i,\ n^*_{\mathrm{h},i-1} \leqslant n_{\mathrm{h}} < n^*_{\mathrm{h},i}; \\ i+1,\ n_{\mathrm{h}} \geqslant n^*_{\mathrm{h},i}, \end{cases}$$

$i \in I$ and $n^*_{\mathrm{h},i}$ represents the target speed to which the i-th subsystem accelerates.

The switching of the i-th subsystem will occur at specified speed n^*_{h}, and the switching instant is recorded as t_i. n^+_{h} represents the high pressure rotor speed at the instant after the switching occurs. So, $n^+_{\mathrm{h}} \geqslant n^*_{\mathrm{h}}$, and the corresponding instant is recorded as t^+_i.

Assumption 1 When switching occurs at any rotational speed, there are $x(t_i) = x(t^+_i)$, $u(t_i) = u(t^+_i)$, and $\sigma(n_{\mathrm{h}}(t^+_i)) = \sigma(n_{\mathrm{h}}(t_{i+1}))$, where $\sigma(n_{\mathrm{h}}(t_{i+1}))$ represents the serial number of the $(i+1)$-th activated subsystem.

In the subsequent text, if not necessary for essential distinction, $\sigma(n_{\mathrm{h}}(t))$ will be written as $\sigma(n_{\mathrm{h}})$. As shown in Fig. 1, when accelerating from point A to point C in the transition state of an aircraft engine, the objective is to find an optimal control input $u^*(t)$ that minimizes the following cost function

$$J(x(t),u(t),\sigma(n_{\mathrm{h}})) = \sum_{\sigma(n_{\mathrm{h}})} L(x(t),u(t),\sigma(n_{\mathrm{h}})) \tag{3}$$

where $L(x(t),u(t),\sigma(n_{\mathrm{h}}))$ is the utility function, which is selected as a quadratic form

$$L(x(t),u(t),\sigma(n_{\mathrm{h}})) = \int_t^{t+\delta t} (x^{\mathrm{T}}Q_{\sigma(n_{\mathrm{h}})}x + u^{\mathrm{T}}R_{\sigma(n_{\mathrm{h}})}u)\mathrm{d}t \tag{4}$$

where $Q_{\sigma(n_{\mathrm{h}})} = Q^{\mathrm{T}}_{\sigma(n_{\mathrm{h}})} \in \Re^{2\times 2}$, $R_{\sigma(n_{\mathrm{h}})} = R^{\mathrm{T}}_{\sigma(n_{\mathrm{h}})} \in \Re$, and δt represents the time interval for collecting system information. The quadratic cost function not only facilitates the rapid increase of high and low pressure rotor speeds during the aircraft engine acceleration process, but also ensures that the fuel flow rate is the optimal solution, which implies a perfect transient performance.

The aim of this paper is to determine the optimal feedback controller $u^* = -K^*_{\sigma(n_{\mathrm{h}})}x$ to minimize the cost function.

According to the traditional linear optimization control theory, we have

$$K^*_{\sigma(n_h)} = R^{-1}_{\sigma(n_h)} B^{\mathrm{T}}_{\sigma(n_h)} P^*_{\sigma(n_h)} \tag{5}$$

where $P^*_{\sigma(n_h)}$ is the unique positive definite solution to the following Lyapunov equation

$$\begin{aligned} &(A_{\sigma(n_h)} - B_{\sigma(n_h)}K_{\sigma(n_h)})^{\mathrm{T}} P_{\sigma(n_h)} + \\ &P_{\sigma(n_h)}(A_{\sigma(n_h)} - B_{\sigma(n_h)}K_{\sigma(n_h)}) - \\ &P_{\sigma(n_h)}B_{\sigma(n_h)}R^{-1}_{\sigma(n_h)}B^{\mathrm{T}}_{\sigma(n_h)}P_{\sigma(n_h)} + Q_{\sigma(n_h)} = 0 \end{aligned} \tag{6}$$

From the form of the Lyapunov equation above, we can think of the solution of the Riccati equation. The obtained $P^*_{\sigma(n_h)}$ allows us to determine $K^*_{\sigma(n_h)}$ from Eq. (5). It is well-known that the Riccati equation is nonlinear with respect to $P_{\sigma(n_h)}$. Besides, in the case of large-dimensional matrices, obtaining its analytical solution is challenging. Hence, we will design a data-driven approach to solve the Riccati equation iteratively for linear switched systems, and a detailed design process is given.

III. Data-Driven Adaptive Optimal Control Strategy with PID Controller and Time-Varying Disturbance Signal

In this section, a data-driven iterative algorithm is proposed for switched linear systems. Instead of the dependence on the system analytical model, the iteration just relies on the information of the state and control quantities in the system operation. To achieve this, one idea is to give an initial control strategy at the beginning of the operation of each subsystem and make it run for a certain period of time to collect the operation data of the system until it meets certain special conditions we set. Then, the optimal feedback control gain for the subsystem is calculated by the data-driven iterative method proposed below.

Based on the switched linear model in Eq. (2), we first employ a control strategy that combines a PID controller and a time-varying disturbance signal at the initial moment of the subsystem operation, namely

$$\begin{aligned} e_{\sigma(n_h)} = -[K_{p_{\sigma(n_h)}}(x(t) + \frac{1}{K_{i_{\sigma(n_h)}}}\int_0^t x(t)\mathrm{d}t + \\ K_{d_{\sigma(n_h)}}\frac{\mathrm{d}x(t)}{\mathrm{d}t})] + e_0 \end{aligned} \tag{7}$$

where $K_{p_{\sigma(n_h)}}$, $K_{i_{\sigma(n_h)}}$, and $K_{d_{\sigma(n_h)}}$ are proportionality coefficient, integration coefficient, and differential coefficient, respectively. $e_0 = a\sin(\omega t)$, where a is the amplitude and ω is the period. $e_{\sigma(n_h)}$ is the exploration noise of the system, whose main purpose is to be used for the online learning process of the subsystem, and acts in the time domain until the specific condition is satisfied, as can be seen in Assumption 2.

The objective is to design an optimal feedback controller for each subsystem to minimize the cost function in Eq. (4), so a controller is formulated as

$$u = -K_{\sigma(n_h)}x + e_{\sigma(n_h)} \tag{8}$$

Then, the closed-loop switched system of Eq. (2) is written as

$$\begin{aligned} \dot{x} = A_{\sigma(n_h)}x + B_{\sigma(n_h)}u = \\ (A_{\sigma(n_h)} - B_{\sigma(n_h)}K_{\sigma(n_h)})x + B_{\sigma(n_h)}e_{\sigma(n_h)} = \\ \hat{A}_{\sigma(n_h)}x + B_{\sigma(n_h)}e_{\sigma(n_h)} \end{aligned} \tag{9}$$

where $\hat{A}_{\sigma(n_h)} = A_{\sigma(n_h)} - B_{\sigma(n_h)}K_{\sigma(n_h)}$.

Utilizing the LQR optimal control theory, Eq. (3) can be written as

$$J(x(t), u(t), \sigma(n_h)) = \sum_{\sigma(n_h)} {x_0}^{\mathrm{T}} P_{\sigma(n_h)} x_0 \tag{10}$$

where $P_\sigma = \int_{t_\sigma}^{t_\sigma+\delta t_\sigma}(e^{\hat{A}^{\mathrm{T}}_{\sigma}\tau}_{\sigma(n_h)}(x^{\mathrm{T}}Q_\sigma x + K^{\mathrm{T}}_\sigma R_\sigma K_\sigma)e^{\hat{A}_\sigma \tau}_{\sigma(n_h)})\mathrm{d}\tau$ and x_0 represents the initial state value of the system.

We propose a new recurrence relation for the switching optimal control problem mentioned in Section II. The optimization of the cost function can be embedded in the optimization of the system value function, which is defined as

$$V(x(t), u(t), \sigma(n_h)) = J(x(t), u(t), \sigma(n_h)) \tag{11}$$

The optimal switching control problem during aircraft engine transient-state acceleration is to find a control input $u(t)$ that minimizes $J(x(t), u(t), \sigma(n_h))$, namely

$$V^*(x(t), u(t), \sigma(n_h)) = \min J(x(t), u(t), \sigma(n_h)) \tag{12}$$

By means of the Bellman optimization criterion, the following recurrence relation can be obtained

$$\begin{aligned} V^*(x(t), u(t), \sigma(n_h(t_i))) = \min L(x(t), u(t), \sigma(n_h(t_i))) + \\ V^*(x(t), u(t), \sigma(n_h(t_{i+1}))) \end{aligned} \tag{13}$$

From Assumption 1, it can be seen that the utility function in Eq. (4) remains constant before and after the subsystem switching, that is

$$\begin{aligned} L(x(t_i), u(t_i), \sigma(n_h(t_i))) = \\ L(x(t_i^+), u(t_i^+), \sigma(n_h(t_i^+))) = \\ L(x(t_i), u(t_i), \sigma(n_h(t_{i+1}))) \end{aligned} \tag{14}$$

Therefore, we have

$$V(x(t), u(t), \sigma(n_h(t_i))) = V(x(t), u(t), \sigma(n_h(t_{i+1}))) \tag{15}$$

When switching occurs during the aircraft engine transition state acceleration, the value function at the switching instant is unchanged. Therefore, $P_{\sigma(n_h(t_k)),k} = P_{\sigma(n_h(t_{k+1})),0}$, where $P_{\sigma(n_h(t_k)),k}$ represents the k-th loop solving $P_{\sigma(n_h)}$ for the i-th subsystem. That is, the final value of subsystem $P_{\sigma(n_h(t_k))}$ before the switching occurs is equal to the initial value of subsystem $P_{\sigma(n_h(t_{k+1}))}$ after the switching occurs. Since only a single subsystem works at a specific moment, in order to minimize the cost function, Eq. (10) is differentiated with respect to time as

$$\begin{aligned} \frac{\mathrm{d}}{\mathrm{d}t}(x^{\mathrm{T}}P_{\sigma(n_h),k}x) = x^{\mathrm{T}}(\hat{A}^{\mathrm{T}}_{\sigma(n_h)}P_{\sigma(n_h),k} + P_{\sigma(n_h),k}\hat{A}_{\sigma(n_h)})x + \\ 2e^{\mathrm{T}}_{\sigma(n_h)}B^{\mathrm{T}}_{\sigma(n_h)}P_{\sigma(n_h),k}x \end{aligned} \tag{16}$$

Hence, the corresponding Lyapunov equation for Eq. (9) is

$$\begin{aligned}&\hat{A}^{\mathrm{T}}_{\sigma(n_{\mathrm{h}})}P_{\sigma(n_{\mathrm{h}}),k}+P_{\sigma(n_{\mathrm{h}}),k}\hat{A}_{\sigma(n_{\mathrm{h}})}+Q_{\sigma(n_{\mathrm{h}})}+\\&K^{\mathrm{T}}_{\sigma(n_{\mathrm{h}}),k}R_{\sigma(n_{\mathrm{h}})}K_{\sigma(n_{\mathrm{h}}),k}=0\end{aligned} \tag{17}$$

Solving Eq. (17) yields a positive definite solution $P_{\sigma(n_{\mathrm{h}}),k}$ of the Lyapunov equation, which corresponds to the strategy evaluation process.

The update results in a new feedback gain, i.e., it corresponds to a strategy enhancement proces

$$K_{\sigma(n_{\mathrm{h}}),k+1}=R^{-1}_{\sigma(n_{\mathrm{h}})}B^{\mathrm{T}}_{\sigma(n_{\mathrm{h}})}P_{\sigma(n_{\mathrm{h}}),k} \tag{18}$$

Combining Eqs. (17) and (18), Eq. (16) can be expressed as

$$\begin{aligned}&\frac{\mathrm{d}}{\mathrm{d}t}(x^{\mathrm{T}}P_{\sigma(n_{\mathrm{h}}),k}x)=\\&-x^{\mathrm{T}}Q_{\sigma(n_{\mathrm{h}}),k}x+2e^{\mathrm{T}}_{\sigma(n_{\mathrm{h}})}R_{\sigma(n_{\mathrm{h}})}K_{\sigma(n_{\mathrm{h}}),k+1}x\end{aligned} \tag{19}$$

where $Q_{\sigma(n_{\mathrm{h}}),k}=Q_{\sigma(n_{\mathrm{h}})}+K^{\mathrm{T}}_{\sigma(n_{\mathrm{h}}),k}R_{\sigma(n_{\mathrm{h}})}K_{\sigma(n_{\mathrm{h}}),k}$.

Through this transformation, the dependency on the model information A and B is eliminated from the cost function.

Then, integrating Eq. (19) over the time interval $[t,t+\delta t]$ results in

$$\begin{aligned}&x^{\mathrm{T}}(t+\delta t)P_{\sigma(n_{\mathrm{h}}),k}x(t+\delta t)-x^{\mathrm{T}}(t)P_{\sigma(n_{\mathrm{h}}),k}x(t)-\\&2\int_t^{t+\delta t}(e^{\mathrm{T}}_{\sigma(n_{\mathrm{h}})}R_{\sigma(n_{\mathrm{h}})}K_{\sigma(n_{\mathrm{h}}),k+1}x)\mathrm{d}t=\\&-\int_t^{t+\delta t}(x^{\mathrm{T}}Q_{\sigma(n_{\mathrm{h}}),k}x)\mathrm{d}t\end{aligned} \tag{20}$$

From Eq. (8), it is evident that

$$\begin{aligned}&e_{\sigma(n_{\mathrm{h}})}{}^{\mathrm{T}}R_{\sigma(n_{\mathrm{h}})}K_{\sigma(n_{\mathrm{h}}),k}x=\\&\left(u+K_{\sigma(n_{\mathrm{h}}),k}x\right)^{\mathrm{T}}R_{\sigma(n_{\mathrm{h}})}K_{\sigma(n_{\mathrm{h}}),k+1}x=\\&x^{\mathrm{T}}K^{\mathrm{T}}_{\sigma(n_{\mathrm{h}}),k}R_{\sigma(n_{\mathrm{h}})}K_{\sigma(n_{\mathrm{h}}),k+1}x+u^{\mathrm{T}}R_{\sigma(n_{\mathrm{h}})}K_{\sigma(n_{\mathrm{h}}),k+1}x\end{aligned} \tag{21}$$

Substituting Eq. (21) into Eq. (20), we obtain

$$\begin{aligned}&x^{\mathrm{T}}(t+\delta t)P_{\sigma(n_{\mathrm{h}}),k}x(t+\delta t)-x^{\mathrm{T}}(t)P_{\sigma(n_{\mathrm{h}}),k}x(t)-\\&2\int_t^{t+\delta t}(x^{\mathrm{T}}K^{\mathrm{T}}{}_{\sigma(n_{\mathrm{h}}),k}R_{\sigma(n_{\mathrm{h}})}K_{\sigma(n_{\mathrm{h}}),k+1}x+\\&u^{\mathrm{T}}R_{\sigma(n_{\mathrm{h}})}K_{\sigma(n_{\mathrm{h}}),k+1}x)\mathrm{d}t=\\&-\int_t^{t+\delta t}(x^{\mathrm{T}}Q_{\sigma(n_{\mathrm{h}}),k}x)\mathrm{d}t\end{aligned} \tag{22}$$

For computational convenience, $P_{\sigma(t_i),k}$ and $K_{\sigma(t_i),k}$ in the iteration process are transformed into a vector. To do this without losing information during the transformation, the traditional method is to use the Kronecker product, denoted as

$$\mathrm{vec}(ABC)=(C^{\mathrm{T}}\otimes A)\mathrm{vec}(B) \tag{23}$$

where A, B, and C are matrices and $\otimes$ indicates the Kronecker product.

Define two functions as

$$\Gamma(\alpha)=\int_\alpha^{\alpha+\delta\tau}(x\otimes x)\mathrm{d}t \tag{24}$$

$$\Pi(\alpha)=\int_\alpha^{\alpha+\delta\tau}(x\otimes u)\mathrm{d}t \tag{25}$$

Furthermore, for any positive integer h, let

$$D_{xx}=\left[x\otimes x|_t^{t+\delta\tau},x\otimes x|_{t+\delta\tau}^{t+2\delta\tau},\ldots,x\otimes x|_{t+(h-1)\delta\tau}^{t+h\delta\tau}\right]^{\mathrm{T}} \tag{26}$$

$$I_{xx}=[\Gamma(t),\Gamma(t+\delta\tau),...,\Gamma((h-1)\delta\tau)]^{\mathrm{T}} \tag{27}$$

$$I_{xu}=[\Pi(t),\Pi(t+\delta\tau),...,\Pi((h-1)\delta\tau)]^{\mathrm{T}} \tag{28}$$

where $D_{xx}\in\Re^{h\times4}$, $I_{xx}\in\Re^{h\times4}$, $I_{xu}\in\Re^{h\times2}$, and $(t,t+h\delta\tau)\in(t,t+\delta t)$.

Combining Eqs. (23)–(28), Eq. (22) can be converted into

$$\begin{aligned}&[D_{xx},-2I_{xx}(I_n\otimes K_{\sigma(n_{\mathrm{h}}),k}R_{\sigma(n_{\mathrm{h}})})-2I_{xu}(I_n\otimes R_{\sigma(n_{\mathrm{h}})})]\times\\&\begin{bmatrix}\mathrm{vec}(P_{\sigma(n_{\mathrm{h}}),k})\\\mathrm{vec}(K_{\sigma(n_{\mathrm{h}}),k+1})\end{bmatrix}=-I_{xx}\mathrm{vec}(Q_{\sigma(n_{\mathrm{h}}),k})\end{aligned} \tag{29}$$

where I_n is the identity matrix.

For the sake of simplicity of expression, denote

$$\begin{aligned}\Psi_k=\Big[&D_{xx},-2I_{xx}(I_n\otimes K_{\sigma(n_{\mathrm{h}}),k}R_{\sigma(n_{\mathrm{h}})})-\\&2I_{xu}(I_n\otimes R_{\sigma(n_{\mathrm{h}})})\Big]\end{aligned} \tag{30}$$

$$\Phi_k=-I_{xx}\mathrm{vec}(Q_{\sigma(n_{\mathrm{h}}),k}) \tag{31}$$

Therefore, Eq. (29) can be expressed as

$$\Psi_k\begin{bmatrix}\mathrm{vec}(P_{\sigma(n_{\mathrm{h}}),k})\\\mathrm{vec}(K_{\sigma(n_{\mathrm{h}}),k+1})\end{bmatrix}=\Phi_k \tag{32}$$

Based on the derivation above, under the given initial control strategy $e_{\sigma(n_{\mathrm{h}})}$, collect data of states and control quantities during online operation, denoted as D_{xx}, I_{xx}, and I_{xu}, and without requiring additional system information. The subsequent iteration process is to calculate Ψ_k and Φ_k.

Assumption 2 The matrices I_{xx} and I_{xu} are the full column rank, satisfying

$$\mathrm{rank}([I_{xx},I_{xu}])=5 \tag{33}$$

Furthermore, we give the steps of the data-driven iterative algorithm based on switched linear systems in Algorithm 1.

Algorithm 1 Optimization transient-state control of the switched linear system for an aircraft engine

Step 1: Find an initial value of K_0 such that $A_{\sigma(n_{\mathrm{h}})}-B_{\sigma(n_{\mathrm{h}})}K_0$ is Hurwitz, and set iteration number $k=0$.

Step 2: The corresponding subsystem is activated, and the initial control strategy $K_0x+e_{\sigma(n_{\mathrm{h}})}$ is employed. Collect system operation data and calculate D_{xx}, I_{xx}, and I_{xu} until the condition in Eq. (33) is satisfied, then stop the data collection process.

Step 3: By solving Eq. (32) to obtain $P_{\sigma(n_{\mathrm{h}}),k}$ and recon-structing the matrix $P_{\sigma(n_{\mathrm{h}}),k}$, such that $P^{\mathrm{T}}_{\sigma(n_{\mathrm{h}}),k}=P_{\sigma(n_{\mathrm{h}}),k}$, the value of $K_{\sigma(n_{\mathrm{h}}),k+1}$ can be obtained.

Step 4: Given the iteration accuracy ε, if $|P_{\sigma(n_{\mathrm{h}}),k}-P_{\sigma(n_{\mathrm{h}}),k-1}|<\varepsilon$, return to step 5, otherwise let $k=k+1$, return to step 3.

Step 5: Use $u=-K_{\sigma(n_{\mathrm{h}}),k}x$ as the new control strategy for the subsystem.

Step 6: If the rotational speed of the subsystem satisfies the designed switching control law under the current control strategy, return to step 1. If not, go to step 5.

IV. Simulation

In this section, experimental data from the transient-state acceleration process of a specific model of aircraft engine are used. Establish a switched linear system model based on multiple steady-state points. The control strategy will use Algorithm 1.

In practical engineering applications, the dynamics of aircraft engines are often described using second-order models [38]. Therefore, the subsystems of the switched linear model are established as

$$A_1=\left[\begin{array}{cc}-0.998,696 & -0.186,637 \\ 2.795,796 & -4.050,835\end{array}\right],\ B_1=\left[\begin{array}{c}102.357,418 \\ 133.704,319\end{array}\right],$$
$$A_2=\left[\begin{array}{cc}-1.265,710 & -0.180,972 \\ 3.349,383 & -4.585,759\end{array}\right],\ B_2=\left[\begin{array}{c}98.602,969 \\ 146.097,848\end{array}\right],$$
$$A_3=\left[\begin{array}{cc}-1.324,523 & -0.172,316 \\ 3.358,687 & -5.133,668\end{array}\right],\ B_3=\left[\begin{array}{c}86.104,469 \\ 170.854,103\end{array}\right],$$
$$A_4=\left[\begin{array}{cc}-1.754,171 & -0.371,982 \\ 4.339,040 & -5.620,150\end{array}\right],\ B_4=\left[\begin{array}{c}116.609,490 \\ 87.041,056\end{array}\right],$$
$$A_5=\left[\begin{array}{cc}-1.710,152 & -0.404,326 \\ 4.245,733 & -6.270,258\end{array}\right],\ B_5=\left[\begin{array}{c}100.541,638 \\ 115.818,841\end{array}\right].$$

The objective is to design a suitable controller to enable the aircraft engine to accelerate from the initial state to the target state. Following the aforementioned principles, the cost function during the acceleration process should be in the following form

$$\begin{aligned}
&J(x(t),u(t),\sigma(n_{\rm h}))= \\
&\int_{t_0}^{t_0+\delta t_0}[(\Delta n_{\rm h}-\Delta n_{\rm h1}^*)^2+(\Delta n_{\rm l}-\Delta n_{\rm l1}^*)^2+\Delta q_{\rm mf}^2]{\rm d}t+ \\
&\int_{t_1}^{t_1+\delta t_1}[(\Delta n_{\rm h}-\Delta n_{\rm h2}^*)^2+(\Delta n_{\rm l}-\Delta n_{\rm l2}^*)^2+\Delta q_{\rm mf}^2]{\rm d}t+ \\
&\int_{t_2}^{t_2+\delta t_2}[(\Delta n_{\rm h}-\Delta n_{\rm h3}^*)^2+(\Delta n_{\rm l}-\Delta n_{\rm l3}^*)^2+\Delta q_{\rm mf}^2]{\rm d}t+ \\
&\int_{t_3}^{t_3+\delta t_3}[\left(\Delta n_{\rm h}-\Delta n_{\rm h4}^*\right)^2+\left(\Delta n_{\rm l}-\Delta n_{\rm l4}^*\right)^2+\Delta q_{\rm mf}^2]{\rm d}t+ \\
&\int_{t_4}^{t_4+\delta t_4}[(\Delta n_{\rm h}-\Delta n_{\rm h5}^*)^2+(\Delta n_{\rm l}-\Delta n_{\rm l5}^*)^2+\Delta q_{\rm mf}^2]{\rm d}t.
\end{aligned}$$

In the implementation of Algorithm 1, during the data collection process in step 2, PID initial controllers tailored to each subsystem are designed. To make the collected data more applicable, a time-varying sinusoidal disturbance signal combined with a PID controller is applied, given by $e_0=0.05\sin(52t)$. The data collection process continued until the condition in Eq. (33) is satisfied, at which time the data collection is stopped, and the algorithm proceeds to step 3. In step 4, the iteration termination condition is set as $\varepsilon=1.0\times10^{-8}$.

Then, we apply the switching control strategy based on Algorithm 1 to the transient-state acceleration process of a certain model of aircraft engine. The iterative calculation trend for solving the feedback control gain for each subsystem is illustrated in Fig. 2. It can be observed that the accuracy requirements are met after approximately 5–7 iterations during the iterative calculation process. The obtained control gains for subsystems are

$$K_1=[0.6541,0.7464],\ K_2=[0.3976,0.9964],$$
$$K_3=[0.2642,1.1235],\ K_4=[0.3429,1.0576],$$
$$K_5=[0.2238,1.0693].$$

The simulated transient-state acceleration process of the aircraft engine in this study involves an increase in high pressure rotor speed from 39,330 to 43,330 r/min and an increase in low pressure rotor speed from 19,920 to 24,220 r/min. In order to verify the superiority of the proposed method, we give two comparison methods. The simulations are conducted for three different control strategies: a single PID controller (each subsystem of the switched linear model established in this article uses a PID controller uniformly), multiple switched PID controllers (each subsystem uses a PID controller respectively), and the control strategy proposed in Algorithm 1. Throughout the entire acceleration process, the time and sequence of subsystem switching are illustrated in Fig. 3. The moment of switching that occurs in the simulation of the proposed algorithm in this paper is earlier compared with other PID control strategies, which demonstrates the faster response of the proposed method.

The response curves of high pressure rotor speed and low pressure rotor speed are shown in Figs. 4 and 5, respectively. Compared with both the single PID controller and the multiple PID controllers, the proposed algorithm accelerates to the target speed more quickly and without overshooting throughout the entire process.

ADRC is a widely applied control method for aircraft engines. We verify the effectiveness of the control strategy proposed in Algorithm 1 through simulation comparison, as shown in Figs. 6 and 7. It is evident that the ADP method significantly outperforms the ADRC controller in terms of response speed and overshoot.

Furthermore, we give the integral of the absolute value of the error criterion (IAE) and the integral of time multiplied by the absolute value of the error criterion (ITAE) indicators of the entire control process, as shown in Tables 1 and 2. IAE can reflect the size of the error (control accuracy), but does not consider the speed of error convergence. ITAE can reflect both the size of the error (control accuracy) and the speed of error convergence, taking into account both the control accuracy and the convergence speed. From Tables 1 and 2, it can be seen that the proposed method is significantly superior to the other two control strategies. The comparisons of simulation results and evaluation indicators show the effectiveness and superiority of the proposed algorithm.

Finally, to highlight the advantages of the algorithm proposed in this paper, the operating data of the aircraft engine working at different altitudes and Mach numbers are obtained and the linear switching system model is established using system identification technology. The proposed algorithm is used for the simulation experiments and the simulation results are shown in Figs. 8 and 9. In addition, to further verify the advantages of the established switching ADP algorithm, the data-driven algorithm of the switching ADP is compared with the conventional ADP data-driven algorithm [27], and the results are shown in Figs. 10 and 11.

V. Conclusion

This study investigates the optimal control problem of the transient acceleration process in aircraft engines and proposes a data-driven optimal control algorithm based on switched

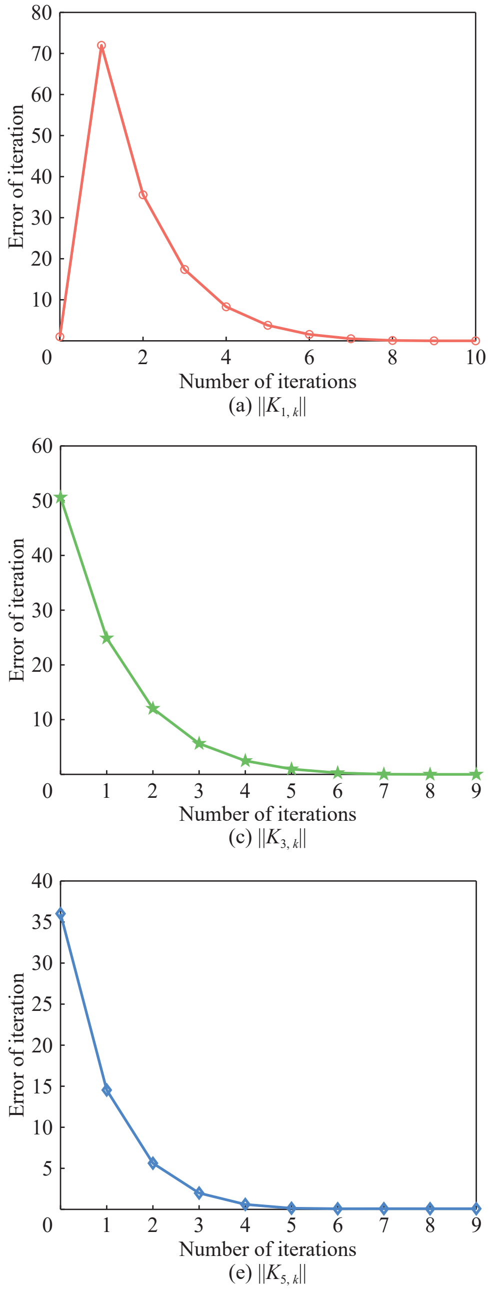

Figure 2 Iterative process of $K = \{K_1, K_2, K_3, K_4, K_5\}$.

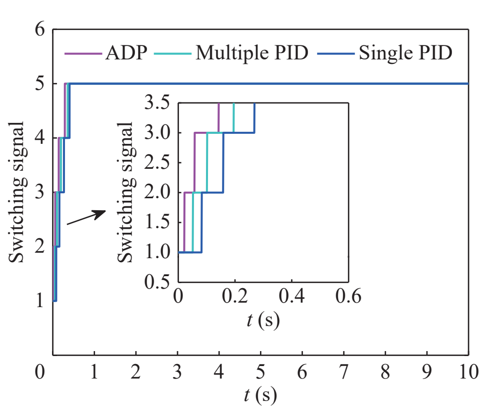

Figure 3 Switching signal of the five subsystems.

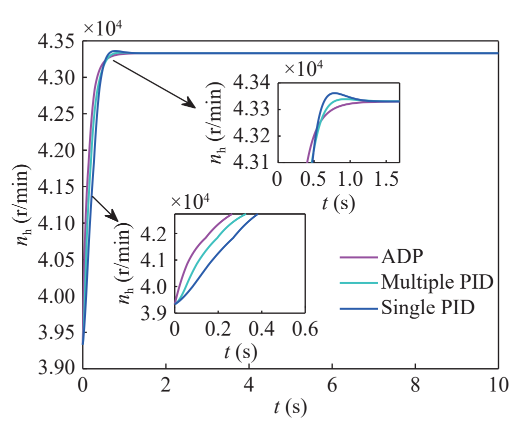

Figure 4 High pressure rotor speed.

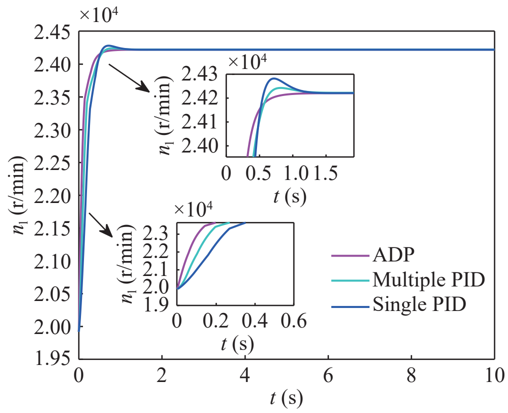

Figure 5 Low pressure rotor speed.

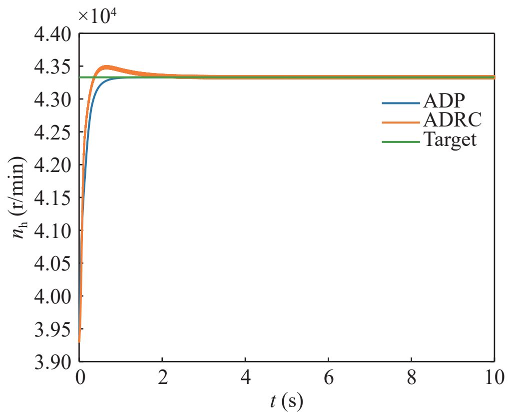

Figure 6 Comparison of high pressure rotor speed between ADP and ADRC.

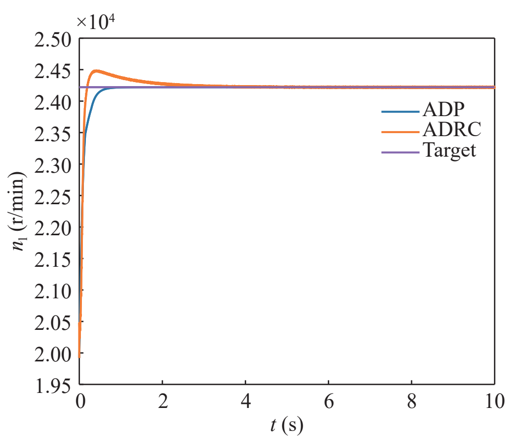

Figure 7 Comparison of low pressure rotor speed between ADP and ADRC.

Table 1 IAE/ITAE indicator for high pressure rotor speed.

Algorithm	IAE	ITAE
ADP	557.1	83.84
Single PID	919.8	160.60
Multiple PID	740.7	179.80

Table 2 IAE/ITAE indicator for low pressure rotor speed.

Algorithm	IAE	ITAE
ADP	428.4	87.69
Single PID	859.7	178.50
Multiple PID	644.0	196.10

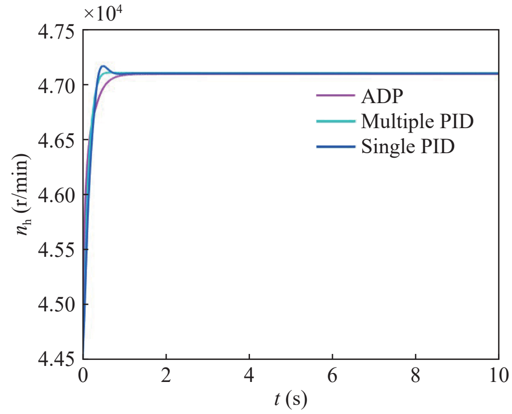

Figure 8 High pressure rotor speed under complex working condition.

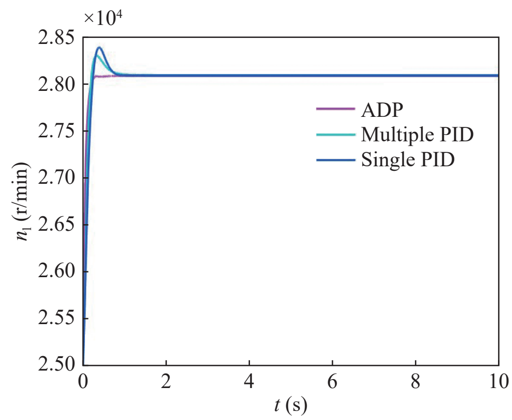

Figure 9 Low pressure rotor speed under complex working condition.

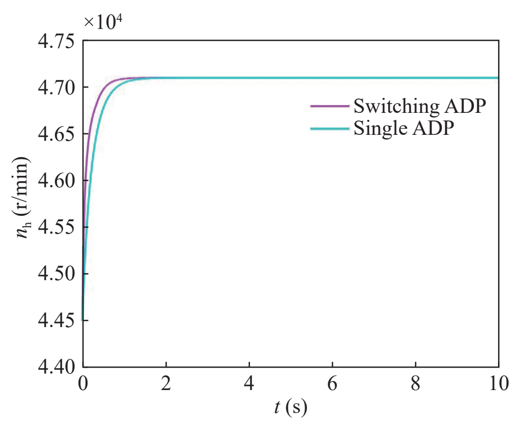

Figure 10 High pressure rotor speed under different data-driven control strategies.

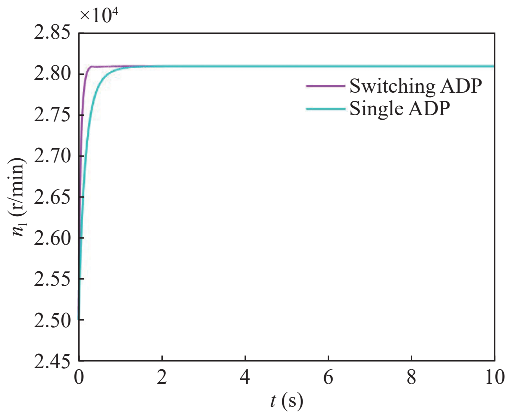

Figure 11 Low pressure rotor speed under different data-driven control strategies.

linear models. First, under a specific initial control strategy, the value function for evaluating system performance indicators is constructed based on the collected operational data. Then, by transforming the optimization problem of the value function into solving the Lyapunov equation, the adaptive dynamic programming algorithm is introduced to iteratively solve for the optimal solution of the Lyapunov equation. Finally, the optimal feedback control gains obtained for each subsystem are applied to the transient acceleration process of aircraft engines based on the switched linear system. The simulation results show that the proposed method exhibits superior performance. Additionally, the data-driven optimal control algorithm does not rely on the system model, making it applicable to unknown switched linear systems and demonstrating strong scalability.

Future work will focus on the following aspects. To study the application of the ADP method in more complex environments and working conditions, the data-driven control method based on disturbance observer is a scientific issue worthy of study in the future. In addition, to demonstrate the universality of this method, the model-free optimal control problem for unknown non-affine nonlinear systems is also worthy of further study.

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Zelin Li received the BS degree in applied physics from Yanshan University, China, in 2022. He is currently pursuing the MS degree at Key Laboratory of Intelligent Control and Optimization for Industrial Equipment (Dalian University of Technology), Ministry of Education, China. His research interests include aero-engine control, adaptive dynamic programming, and data-driven control.

Yan Shi received the BS degree in automation and the PhD degree in control theory and control engineering from Northeastern University, China, in 2013 and 2019, respectively. He is currently an associate professor at Key Laboratory of Intelligent Control and Optimization for Industrial Equipment (Dalian University of Technology), Ministry of Education, China. His research interests include switched systems, switching control, bumpless transfer control, and aero-engine control.

Shuoshuo Liu received the BS degree from Northwestern Polytechnical University, China, in 2018. He is currently pursuing the PhD degree at Key Laboratory of Intelligent Control and Optimization for Industrial Equipment (Dalian University of Technology), Ministry of Education, China. His research interests include data-driven control, system identification, and aero-engine control.

Tao Sun received the MS degree in operational research and cybernetics from School of Mathematical Sciences, Dalian University of Technology, China, in 2017, and the PhD degree in control theory and control engineering from School of Control Science and Engineering, Dalian University of Technology, China, in 2022. In 2024, he worked as a postdoctoral fellow at Department of Automation, Tsinghua University, China. He is currently an associate professor at School of Control Science and Engineering, Dalian University of Technology, China. His research interests include aero-engine control and optimization.

Xudong Zhao received the BS degree in automation from Harbin Institute of Technology, China, in 2005, and the PhD degree from Control Science and Engineering from Space Control and Inertial Technology Center, Harbin Institute of Technology, China, in 2010. He was a lecturer and an associate professor at China University of Petroleum, China. From March 2013, he was at Bohai University, China, as a professor. In 2014, he worked as a postdoctoral fellow at Department of Mechanical Engineering, The University of Hong Kong, China. Since December 2015, he has been with Dalian University of Technology, China, where he is currently a professor. He serves as associate editor for *IEEE Transactions on Systems, Man, and Cybernetics: Systems*, *Nonlinear Analysis: Hybrid Systems*, *Neurocomputing*, *International Journal of General Systems*, *ACTA Automatica Sinica*, *Assembly Automation*, *Journal of Aeronautics*, etc. In addition, he was awarded as the 2017–2020 Web of Science Highly Cited Researcher in Engineering. His research interests include hybrid systems, positive systems, multi-agent systems, and control of aero-engine.

Lijuan Liu received the MS degree in computer engineering from Liaoning Normal University, China, in 2004, and the PhD degree in control theory and control engineering from Dalian University of Technology, China, in 2019. She was a lecturer in 2006 and an associate professor in 2019 at School of Software, Dalian Jiaotong University, China. She is currently a professor at Dalian Jiaotong University, China. Her research interests include switching systems, time delay systems, positive systems, and adaptive control.