A Brief Overview Study of Open Quantum System for Application in Human-Machine Interaction

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Abstract—**Human-machine interaction can incorporate several components which when suitably designed, synthesized, and integrated can benefit the human decision maker in performing various tasks and activities. These interactions can be designed through multiple modalities such as textural, graphic, audio, or any tangible interfaces depending on the specific nature of the decision-making task which needs to be performed by human. The control and decision-making architecture of such human-machine interaction can be defined in variety of ways. In this paper, we present an overview of an architecture which has been recently proposed in the literature based on the notion of open quantum systems. We present a brief interpretation of some of the main modelling and solution components of this framework. The main implication of this framework is that it presents the state of the decision maker and the interacting environment in a probabilistic framework, where the time evolution of this ensemble is estimated within the interpretation and solutions of the open quantum system.**

Index Terms—**Quantum model, open system, human-machine interaction, master equation**

I. Introduction

The advent of quantum technology and its various interpretations have opened new frontiers in various fields, including computational sensing and information technology. Among these, the application of open quantum systems that interact with their external environments may offer a unique interpretation and capabilities for various system modelling. This paper presents how such system has been utilized in the literature to leverage design of monitoring systems for human-machine interaction, for example, in human-in-the-loop (HITL) interface with ambient guidance system aimed in performing activities. By harnessing the unique modelling and data processing capabilities of open quantum systems, it may be possible to explore various approaches to enhance autonomy of users, particularly those with movement and activity decision challenges.

In quantum mechanics, an open system refers to a quantum mechanical system that interacts with its external environment [1, 2]. This interaction leads to the exchange of energy and information between the system and its surroundings. As a result, the dynamics of the system is influenced by these external factors. Unlike closed quantum systems, which evolve in isolation, open quantum systems are influenced by external factors, leading to complex dynamics. This theory, traditionally used in physics, is now being explored for applications in modelling human cognition, especially in situations involving decision-making and interaction with dynamic environments [3–5]. It is shown experimentally that human decision making processes are not easily explained by classical theories. People can hold multiple, sometimes contradictory thoughts simultaneously, and their decisions can be influenced by various external factors in ways that are probabilistic rather than deterministic. Quantum open system theory has been explored in the literature as a possible framework for modelling complex human behaviours, particularly in scenarios where humans interact with their environment in unpredictable ways. In applications such as decision-making or movement guidance, human behavior often exhibits non-classical features such as uncertainty, context-dependence, and emergent phenomenon. Quantum open system theory can be interpreted and configured to capture these complexities by treating humans as open quantum systems interacting with their surroundings.

II. Human-Machine Interaction

In control and automation literature, human-machine interaction, or human-in-the-loop in decision making is defined as a framework for creating multi-modal guidance that integrates human judgement and decision while performing various tasks, movements, or activities. Such integration with the controlled monitoring system can enhance the quality and effectiveness of outcomes of the decisions. In this paradigm, humans are not just passive agents within the system but active participants who through their monitoring of their movements and activities, provide feedback to the guidance system. For example, the role of human operator in advancements in Industry 4.0 is studied, which aims to establish smart factories with enhanced automation in supply chains and production lines [6]. Ensuring safe work environments for these human-robot collaborations is critical. The unpredictability of human behaviour and its impact on work-flow execution introduce significant safety challenges. The research presents an advanced formal verification methodology. This methodology incorporates a non-deterministic model of operator behaviours to identify and mitigate hazardous situations caused by human errors. Human behaviour in seeking control over their advertising experiences is studied [7]. The study proposes a novel framework which includes product involvement, intrusiveness, time watching the advertisement, and recall, to increase advertising effectiveness through consumer empowerment. Conversely, perceived intrusiveness leads to

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increased advertisement-skipping behaviour and negatively impacts advertisement and brand dissemination. Recent developments in recommender systems are reviewed in Ref. [8], highlighting the major challenges and evaluating the existing algorithms. It also explores the physical aspects to understand the macroscopic behaviour of these systems. The paper discusses the potential impacts and future directions, emphasizing that recommender systems have deep scientific roots and intersect various research domains, making them appealing to physicists and interdisciplinary researchers alike. A game-theoretic model is analysed, where individuals compete over shared, failure prone resources [9]. Findings reveal that risk preferences can produce unexpected outcomes. In Ref. [10], a framework is proposed, which monitors both human and machine states to provide appropriate feedback to the human operator beside only monitoring the state of the machine. The work demonstrates how partially observable Markov decision process (POMDP) can serve as a unified framework for integrating the human model, machine dynamic model, and observation model for human-in-the-loop decision systems. A human-assisted robotic planning and sensing framework for active semantic sensing and planning in human-robot teams is proposed [11]. The proposed framework integrates online sampling based POMDP policies, multi-modal human-robot interaction, and Bayesian data fusion. Dynamic model updating enables robotic agents to actively query humans for novel and relevant semantic data, improving model accuracy and state beliefs for enhanced online planning.

Within the human-machine interaction paradigm is the emerging field of ambient intelligence and persuasive technology, which merges information technology with cognitive science, creating smart environments that can intelligently respond to human actions and even influence behaviour. Ambient guidance refers to the unobtrusive and context-aware assistance provided to individuals as they move through and interact with their environments. This can be particularly useful in various settings, such as smart homes, hospitals, airports, and urban spaces. A real-time vision based system is designed to assist individuals with dementia in hand-washing [12]. Utilizing only video inputs, the system provides verbal or visual prompts or engages a human caregiver when needed. It combines a Bayesian sequential estimation framework for tracking hands and towels with a decision-theoretic framework using a partially observable Markov decision process for action policies. An ambient intelligent system is designed to interface with the person by directly providing reminders and advice to individuals with dementia [13]. It utilizes real-time activity monitoring to offer reminders and warnings, ensuring users to maintain healthy routines. A smart home system tailored to the unique requirements of people with dementia and their caregivers is developed [14]. By collecting and analysing stakeholder requirements, the study proposes extensions and adaptations to existing technologies, resulting in a prototype designed specifically for dementia care. Various aspects of smart home multi-modal auto-prompting are investigated for enhancing the functional independence of persons with dementia [15]. It is observed that effective auto-prompting systems must consider various contextual factors, including cognitive impairment level, task familiarity, environmental familiarity, and the interaction between the person and the prompting system, particularly when stress is observed, highlighting the potential need for human communication [16].

In human-in-the-loop decision-making or ambient movement guidance systems, humans are influenced by their environments, which can be dynamic, uncertain, and highly interactive. Quantum open system theory can provide an approach to account for influences of the environment on human behaviour. By modelling humans as open systems that interact with their surroundings, it is possible to capture how environmental factors affect decision-making processes or guide human movement in real-time [3, 5]. Recent research shows that certain aspects of human cognition, such as decision-making under uncertainty or information processing in complex environments, can exhibit quantum-like behavior [4]. More recently, the work in Ref. [17] demonstrates through stochastic Lyapunov arguments that through a design of external decision making controller, it is possible to enable the human-side of decision making process, modelled as open quantum system, to converge to an optimal decision solution. Following previous contributions of the author in review of applications of Bayes and quantum probability in various decision making frameworks [18–20], this paper presents a brief overview study of open quantum system in decision making for human-machine interaction.

The rest of this paper is organized as follows. Section III presents definition of the open quantum system and definition of the density operator for an open composite system. It also presents how the density operator of the individual subsystem defined by mixed operator can be extracted using the trace operator. Section IV presents how the definition of Hamiltonian, which is used in classical mechanics to define the time evolution of the state, is extended to the time evolution of the density operator in open quantum system. Section V presents the main equation which can be used to solve for such continuous time evolution and also a method which can be followed to solve for its discrete time evolution. Section VI presents a simple walk-through example of how some of the highlighted modelling and solutions can be implemented followed by Section VII with concluding remarks.

III. Open Quantum System

In quantum mechanics, an open system refers to a quantum mechanical system that interacts with its external environment [1, 2]. This interaction leads to, for example, the exchange of energy or information between the system and its surroundings. As a result, the dynamics of the system defined by its state $|\Psi\rangle$ is influenced by these external factors.

The state of the system can be defined as (pure state) $|\Psi\rangle$ in a Hilbert space $\mathcal{H}$ with $|i\rangle \in \mathcal{H}$ as its i-th basis vector. For example, let $|\Psi\rangle = \alpha|i\rangle + \beta|j\rangle$, where $|i\rangle$ and $|j\rangle$ can be the two dimensional basis vectors in this space and the coefficients satisfy the following identity: $\|\alpha\|^2 + \|\beta\|^2 = 1$.

The quantum state can also be defined in terms of a density matrix (or density operator). This representation can be obtained through definition of the pure state description based on its quantum state $|\Psi\rangle$. This is obtained through the outer product of the pure state vector with itself, i.e., $\rho = |\Psi\rangle\langle\Psi|$ with identity for the result of the trace operation of this matrix, i.e., $\mathrm{Tr}(\rho) = 1$ [21]. For example, using the above definition, define a pure state $|\Psi\rangle = 0.707|i\rangle + 0.707|j\rangle$, and the density operator in terms of this pure state description can be written as

$$\rho = |\Psi\rangle\langle\Psi| = \begin{pmatrix} 0.707 \\ 0.707 \end{pmatrix}\begin{pmatrix} 0.707 & 0.707 \end{pmatrix}\begin{pmatrix} 0.500 & 0.500 \\ 0.500 & 0.500 \end{pmatrix}.$$

Density operator can also be defined in mixed state meaning that the system is described by a statistical ensemble of basis pure of states with different probabilities. The density matrix is a sum of outer products of the basis weighted by these probabilities, i.e., $\rho = \sum_i \mathrm{Pr}_i |\psi_i\rangle\langle\psi_i|$, where $|\psi_i\rangle$ is the pure state and Pr_i is the probability associated with the pure state with the property of $\sum_i \mathrm{Pr}_i = 1$ [21]. For example, consider a system which is in the pure state $|i\rangle$ with a probability of $\mathrm{Pr}_i = 0.7$ and in the pure state $|j\rangle$ with a probability of $1 - \mathrm{Pr}_i = 1 - 0.7 = 0.3$, the density matrix can be defined as

$$\rho = 0.7|i\rangle\langle i| + 0.3|j\rangle\langle j| = \begin{pmatrix} 0.7 & 0.0 \\ 0.0 & 0.3 \end{pmatrix}.$$

An open composite system refers to a scenario where multiple quantum systems are interacting with each other and with their external environments. For an open composite system, the density matrix evolves over time due to the interactions with the environment.

A composite system consisting of two constituents, M (the system of interest) and E (the environment), is defined [5, 21, 22]. The mixed state of this composite system as density operator can be defined as $\rho_c = \rho_{(M+E)}$. This density matrix captures the joint state of both the system M and the environment E which can be written as a tensor product of the density matrices representing the states of M and E individually, or $\rho_c = \rho_{(M+E)} = \rho_M \otimes \rho_E$, where ρ_M and ρ_E represent the density matrices of the systems M and E respectively and $\otimes$ represents the tensor product, a mathematical operation used to combine two quantum systems into a single composite system. The tensor product has several key properties that make it suitable for describing composite systems. For example, if ρ_M is an $n \times n$ matrix and ρ_E is an $m \times m$ matrix, the tensor product $\rho_M \otimes \rho_E$ is an $(n \cdot m) \times (n \cdot m)$ matrix. Let ρ_M and ρ_E be

$$\rho_M = \begin{pmatrix} p & 0 \\ 0 & 1-p \end{pmatrix},$$

$$\rho_E = \begin{pmatrix} q & 0 \\ 0 & 1-q \end{pmatrix},$$

where p represents a probability of being at a state defined by ρ_M and q represents a probability of being at a state defined by ρ_E. We can compute $\rho_c = \rho_{(M+E)} = \rho_M \otimes \rho_E$ as

$$\rho_c = \begin{pmatrix} p\begin{pmatrix} q & 0 \\ 0 & 1-q \end{pmatrix} & 0\begin{pmatrix} q & 0 \\ 0 & 1-q \end{pmatrix} \\ 0\begin{pmatrix} q & 0 \\ 0 & 1-q \end{pmatrix} & (1-p)\begin{pmatrix} q & 0 \\ 0 & 1-q \end{pmatrix} \end{pmatrix}.$$

The mixed state of one of the quantum systems of the composite system, for example system M, can be obtained from the description of the composite state by performing a partial trace over the degrees of freedom of the other system, for example, the environment E [21]. This effectively traces out the environment from the composite density matrix and leaves the reduced density matrix describing only the system M. Mathematically, this is expressed as $\rho'_M = \mathrm{Tr}_E \rho_{(M+E)}$. The trace operation Tr_E is performed by summing over all possible states of the environment E while keeping the states of system M fixed. For example, represent the density matrix $\rho_{(M+E)}$ in a basis that diagonalizes the environment degrees of freedom, then the trace operation simplifies to $\rho'_M = \sum_k \langle k|_E \rho_{(M+E)} |k\rangle_E$, where $|k\rangle_E$ represents the basis state of the environment. This reduced density matrix captures all the information about the state of system M while averaging over the possible states of the environment E. Using the above example of the composite system consisting of two two-dimensional Hilbert spaces, the following basis of the composite density matrix can be written as

$$\{|i\rangle_M \otimes |i\rangle_E, |i\rangle_M \otimes |j\rangle_E, |j\rangle_M \otimes |i\rangle_E, |j\rangle_M \otimes |j\rangle_E\}.$$

The partial trace of E involves summing the diagonal elements corresponding to degrees of freedom of E for each state of M, or $\rho'_M[i,j] = \sum_k \langle k|_E \rho_c |k\rangle_E$, where $|k\rangle_E$ represents the state of E ($|i\rangle_E$ and $|j\rangle_E$). For $|i\rangle_M$, diagonal element for E for $|i\rangle_M$, or entry $(0,0)$, is $pq + p(1-q) = p$; and for $|j\rangle_M$, or entry $(1,1)$, is $(1-p)q + (1-p)(1-q) = 1-p$. Off-diagonal terms for this example are all zero.

IV. Hamiltonian

In the classical mechanics, the Hamiltonian H is defined as the sum of the kinetic energy T and the potential energy V of a system (i.e., total energy), or $H = T + V$ [23, 24]. In the classical description, T is associated with the motion of the system, and V is associated with the forces acting on the system. This definition of total energy of the system combined with the Hamilton's equation of motion allows modelling and prediction of future state trajectories of classical objects using relationships $\mathrm{d}q/\mathrm{d}t = \partial H/\partial p$ and $\mathrm{d}p/\mathrm{d}t = -\partial H/\partial q$, where q represents generalized coordinate (e.g., position), p represents generalized momenta (e.g., momentum), and $\mathrm{d}q/\mathrm{d}t$ and $\mathrm{d}p/\mathrm{d}t$ represent the time rates of change of these variables over time.

In quantum mechanics, the Hamiltonian also represents the total energy of a system [21]. It is an operator (Hermitian operator) that describes the dynamics of a quantum system. The Hamiltonian operator is essential for understanding how a quantum system evolves over time. It is an operator that governs the evolution of a quantum state $|\Psi(t)\rangle$ over time through solution of the Schrödinger's equation. However, determination of the Hamiltonian of a continuous quantum system is very difficult and for most applications of the quantum mechanics to the field of quantum information and computation, the representation of a Hamiltonian for discrete evaluation of the quantum state is used. For example, the solution to Schrödinger's equation is suggested as [21]

$$|\Psi(t_2)\rangle = \mathrm{e}^{-iH(t_2-t_1)}|\Psi(t_1)\rangle = U(t_1,t_2)|\Psi(t_1)\rangle \tag{1}$$

where H is the Hamiltonian of the system and $U(t_1,t_2)$ is a unitary operator. Furthermore, any unitary operator such as U, can be defined as $U=\exp(iK)$ for Hermitian operator K. As a result, it is possible to establish a relationship between the Hamiltonian H of a continuous system and the Hermitian operator in a discrete-time description, using unitary operators. In other words, in quantum mechanics, Hamiltonian of the system can be defined by a Hermitian operator. Using this property of Hamiltonian in Eq. (1), it will result in the definition of $U(t_1,t_2)$ which has unitary properties. It ensures that the evolution of the quantum state preserves the probabilities (or the "norm") of the state, which is essential for the consistency of quantum mechanics. A unitary operator U has the property that $U^\dagger U = I$, where $U^\dagger$ is the conjugate transpose of U, and I is the identity operator. This property ensures that the operator preserves the inner product of states, meaning that it does not change the total probability.

Hermitian operators, also known as self-adjoint operators, have several important properties that make them fundamental in quantum mechanics. For example, they have real eigenvalues, orthogonal eigenfunctions, preservation of inner product, and self-adjoint property, which means that the operator is equal to its own adjoint.

Using the above example of an open composite system consisting of M and E, a simple representation of the Hamiltonian of the composite system H_c can be written as [25]

$$H_\mathrm{c} = H_M \otimes I_E + I_M \otimes H_E,$$

where H_M and H_E are the Hamiltonians of the system and environment respectively, H_c is the Hamiltonian of the composite system, and I is the identity matrix. As a simplified representation of the Hamiltonian, we have

$$H_M = \begin{bmatrix} 0.5 & 0.0 \\ 0.0 & -0.5 \end{bmatrix},$$

$$H_E = \begin{bmatrix} 0.3 & 0.0 \\ 0.0 & -0.3 \end{bmatrix},$$

which can result in the description of the Hamiltonian of the composite as

$$H_\mathrm{c} = \begin{bmatrix} 0.5 & 0.0 \\ 0.0 & -0.5 \end{bmatrix} \otimes I + I \otimes \begin{bmatrix} 0.3 & 0.0 \\ 0.0 & -0.3 \end{bmatrix} = \begin{bmatrix} 0.8 & 0.0 & 0.0 & 0.0 \\ 0.0 & 0.8 & 0.0 & 0.0 \\ 0.0 & 0.0 & -0.8 & 0.0 \\ 0.0 & 0.0 & 0.0 & -0.8 \end{bmatrix}.$$

V. Master Equation

In general cases, we do not know the state of the system $|\Psi\rangle$ and it needs to be described in a form of density matrix/operator ρ as described above. The natural extension of the Schrödinger equation which describes the evaluation of the quantum state which can now operate on the density matrix leads to the Von Neumann equation. This equation describes the time evolution of density matrix in quantum mechanics and it is defined by [2, 21]

$$i\hbar\frac{\mathrm{d}\rho}{\mathrm{d}t} = [H,\rho] \tag{2}$$

where $\hbar$ is the reduced Planck constant. $[H,\rho] = H\rho - \rho H$ is a commutator operator between the Hamiltonian representation of the system and the density operator. It captures the influence of the Hamiltonian H on the density matrix ρ and vice versa. If it is zero, it implies that the Hamiltonian and the density matrix commute, indicating a specific relationship between the dynamics of the system and its quantum state evolution. The commutator is essential for understanding how the density matrix evolves over time in response to the dynamics of the system governed by the Hamiltonian H. It encapsulates the non-commutative nature of quantum mechanics, where the order of operations matters and influences the outcome.

Solving the Von Neumann equation yields the time evolution of the density matrix $\rho(t)$, providing information about the statistical ensemble of quantum states and the quantum uncertainties of the system over time. In the above equation, the reduced Planck constant $\hbar$ is a fundamental physical constant. When the reduced Planck constant is set to 1, it typically means that other units are often scaled accordingly. This is a common choice in theoretical discussions and calculations where simplification is desired [21].

The quantum master equation is a fundamental equation in quantum mechanics that describes the time evolution of the density matrix of an open quantum system (e.g., composite system). It provides a way to study how the state of a quantum system changes over time due to interactions with its environment. The general form of the quantum master equation (i.e., Lindblad master equation) is given by [26]

$$\frac{\mathrm{d}\rho_\mathrm{c}}{\mathrm{d}t} = -i[H_\mathrm{c},\rho_\mathrm{c}] + \mathcal{D}(\rho_\mathrm{c}) \tag{3}$$

where H_c is the Hamiltonian of the system and $\mathcal{D}(\rho_\mathrm{c})$ represents the dissipative term, which describes the influence of the environment on the system.

The dissipative term $\mathcal{D}(\rho_\mathrm{c})$ is often written in the Lindblad form as [2]

$$\mathcal{D}(\rho_\mathrm{c}) = \sum_k \left(L_k \rho_\mathrm{c} L_k^\dagger - \frac{1}{2}\left\{ L_k^\dagger L_k, \rho_\mathrm{c} \right\} \right) \tag{4}$$

where L_k is the arbitrary Lindblad operator, which describes the interaction between the system and its environment. $\{A,B\}$ denotes the anti-commutator of operators A and B defined as $\{A,B\} = AB + BA$, which measures how symmetrically two operators A and B are. $(\cdot)^\dagger$ represents the Hermitian conjugate operation.

In above equation, the interaction between the system and its environment is assumed to be time-independent or, at least, varies slowly enough compared to the time-scale of the system dynamics. This assumption combined with the assumption of memoryless system (i.e., Markov assumption) makes the equation easier to solve. It allows the description of the evolution of the system using differential equations that only involve the current state of the system and its immediate environment, without explicitly tracking the entire history of past interactions.

There exist variety of approaches which can be followed for solving Eq. (3). One common approach involves numerical integration, where the time evolution is discretized, and techniques like Runge-Kutta are used to solve Eq. (3) iteratively [27] or through solutions by vectorization of the state, as detailed in Refs. [3, 5, 28]. Antoher appraoch is through usage of Kraus operators approach [29], which involves discretizing the time evolution and representing the evolution of the system via a set of Kraus operators where the Lindblad master equation can be seen as the continuous limit of this formalism. In the following, we briefly highlight the method based on Kraus operators [2, 17, 27].

In Kraus operators approach, the evolution of the density matrix, Eq. (3), over a small time step Δt, is written as

$$\rho_c(t+\Delta t) = K_0\rho_c(t)K_0^\dagger + \sum_i K_i\rho_c(t)K_i^\dagger \tag{5}$$

where K_0 and K_i are Kraus operators defined as

$$\begin{aligned} K_0 &= I - H_c\Delta t - \frac{1}{2}\sum_i L_i^\dagger L_i \Delta t, \\ K_i &= \sqrt{\Delta t} L_i \end{aligned} \tag{6}$$

where L_i is Lindblad operator.

Once the Kraus operators are constructed for a single time step Δt, the density matrix can be updated iteratively for successive time steps. In the limit $\Delta t \to 0$, the discrete evolution described by the Kraus operators converges to the continuous-time evolution described by the Lindblad master equation. This shows the equivalence between the Kraus operator formalism and the Lindblad master equation. The Kraus operators approach is especially useful when simulating open quantum systems numerically, as the evolution can be computed step by step [21].

VI. Exploratory Example

In the following and through a simple numerical example, we demonstrate some aspects of the above framework and solution which are proposed in Refs. [3–5, 17]. Here again let M represent the human decision state which we assume to include beliefs, preferences, and intentions. We also use E to represent the external ambient guiding system, which can include environmental stimuli, information cues, and guidance mechanisms.

Decision maker is chosen between two movements: $|L\rangle$ for moving left and $|R\rangle$ for moving right. It is assumed that the ambient environment can affect the decision-making process and has two possible states $|E_1\rangle$ for neutral (no strong influence) and $|E_2\rangle$ for external guidance or stimulus. Let the following density matrices be defined at an initial condition when $t = 0$ as

$$\rho_M(0) = \begin{bmatrix} 0.6 & 0.0 \\ 0.0 & 0.4 \end{bmatrix}, \quad \rho_E(0) = \begin{bmatrix} 0.7 & 0.0 \\ 0.0 & 0.3 \end{bmatrix},$$

or

$$\rho_c(0) = \rho_M(0) \otimes \rho_E(0) = \begin{bmatrix} 0.42 & 0.00 & 0.00 & 0.00 \\ 0.00 & 0.18 & 0.00 & 0.00 \\ 0.00 & 0.00 & 0.28 & 0.00 \\ 0.00 & 0.00 & 0.00 & 0.12 \end{bmatrix},$$

which also satisfy the trace conditions of the probability density, $\mathrm{Tr}(\rho_M) = 1$, $\mathrm{Tr}(\rho_E) = 1$, and $\mathrm{Tr}(\rho_c) = \mathrm{Tr}(\rho_M \otimes \rho_E) = 1$.

In the following, we ignore the first part of Eq. (3) which represents the evolution of the density matrix due to the Hamiltonian operator and only focus on the dissipative term of the equation by defining the three terms of the Lindblad operator L_k as [3, 27, 30]

$$L_1 = \begin{bmatrix} 0.6 & 0.0 \\ 0.0 & 0.3 \end{bmatrix} \otimes I_E,$$
$$L_2 = \begin{bmatrix} 0.4 & 0.0 \\ 0.0 & 0.7 \end{bmatrix} \otimes I_E,$$
$$L_3 = \begin{bmatrix} 0.5 & 0.5 \\ 0.5 & 0.5 \end{bmatrix} \otimes \begin{bmatrix} 0.8 & 0.2 \\ 0.2 & 0.8 \end{bmatrix}.$$

Let the discrete time-step evaluation of the system be $\Delta t = 1$. The Kraus operators defined in Eq. (6) in terms of Lindblad operators described in Eq. (5) can be written as

(1) K_1 from L_1

$$K_1 = \sqrt{1}\begin{bmatrix} 0.6 & 0.0 \\ 0.0 & 0.3 \end{bmatrix} \otimes I_E = \begin{bmatrix} 0.6 & 0.0 \\ 0.0 & 0.3 \end{bmatrix} \otimes I_E,$$

(2) K_2 from L_2

$$K_2 = \sqrt{1}\begin{bmatrix} 0.4 & 0.0 \\ 0.0 & 0.7 \end{bmatrix} \otimes I_E = \begin{bmatrix} 0.4 & 0.0 \\ 0.0 & 0.7 \end{bmatrix} \otimes I_E,$$

(3) K_3 from L_3

$$K_3 = \sqrt{1}\begin{bmatrix} 0.5 & 0.5 \\ 0.5 & 0.5 \end{bmatrix} \otimes \begin{bmatrix} 0.8 & 0.2 \\ 0.2 & 0.8 \end{bmatrix},$$

where we exclude K_0 operator which represents the Hamiltonian contribution to the system evolution [1].

The simplified expression for the evolution of the composite density matrix is then written as $\rho_c(t+\Delta t) = \sum_i K_i\rho_c(t)K_i^\dagger$. Using the definition of K_i, each component within the summation can be computed. For example, for the term containing K_1 term, we have

$$K_1\rho_c(0)K_1^\dagger = \begin{bmatrix} 0.60 & 0.00 & 0.00 & 0.00 \\ 0.00 & 0.60 & 0.00 & 0.00 \\ 0.00 & 0.00 & 0.30 & 0.00 \\ 0.00 & 0.00 & 0.00 & 0.30 \end{bmatrix} \cdot \begin{bmatrix} 0.42 & 0.00 & 0.00 & 0.00 \\ 0.00 & 0.18 & 0.00 & 0.00 \\ 0.00 & 0.00 & 0.28 & 0.00 \\ 0.00 & 0.00 & 0.00 & 0.12 \end{bmatrix} \cdot \begin{bmatrix} 0.60 & 0.00 & 0.00 & 0.00 \\ 0.00 & 0.60 & 0.00 & 0.00 \\ 0.00 & 0.00 & 0.30 & 0.00 \\ 0.00 & 0.00 & 0.00 & 0.30 \end{bmatrix},$$

K_2

$$K_2\rho_c(0)K_2^\dagger = \begin{bmatrix} 0.40 & 0.00 & 0.00 & 0.00 \\ 0.00 & 0.40 & 0.00 & 0.00 \\ 0.00 & 0.00 & 0.70 & 0.00 \\ 0.00 & 0.00 & 0.00 & 0.70 \end{bmatrix} \cdot \begin{bmatrix} 0.42 & 0.00 & 0.00 & 0.00 \\ 0.00 & 0.18 & 0.00 & 0.00 \\ 0.00 & 0.00 & 0.28 & 0.00 \\ 0.00 & 0.00 & 0.00 & 0.12 \end{bmatrix} \cdot \begin{bmatrix} 0.40 & 0.00 & 0.00 & 0.00 \\ 0.00 & 0.40 & 0.00 & 0.00 \\ 0.00 & 0.00 & 0.70 & 0.00 \\ 0.00 & 0.00 & 0.00 & 0.70 \end{bmatrix},$$

and K_3

$$K_3\rho_c(0)K_3^\dagger = \begin{bmatrix} 0.12 & 0.03 & 0.12 & 0.03 \\ 0.03 & 0.12 & 0.03 & 0.12 \\ 0.12 & 0.03 & 0.12 & 0.03 \\ 0.03 & 0.12 & 0.03 & 0.12 \end{bmatrix} \cdot \begin{bmatrix} 0.42 & 0.00 & 0.00 & 0.00 \\ 0.00 & 0.18 & 0.00 & 0.00 \\ 0.00 & 0.00 & 0.28 & 0.00 \\ 0.00 & 0.00 & 0.00 & 0.12 \end{bmatrix} \cdot \begin{bmatrix} 0.12 & 0.03 & 0.12 & 0.03 \\ 0.03 & 0.12 & 0.03 & 0.12 \\ 0.12 & 0.03 & 0.12 & 0.03 \\ 0.03 & 0.12 & 0.03 & 0.12 \end{bmatrix}.$$

The solution for the composite density matrix ρ_c at next time step of $t+\Delta t$ when starting from the initial condition of $t=0$ can be obtained as

$$\rho_c(t+\Delta t) = \begin{bmatrix} 0.3306 & 0.0039 & 0.0114 & 0.0039 \\ 0.0039 & 0.1422 & 0.0039 & 0.0054 \\ 0.0114 & 0.0039 & 0.1736 & 0.0039 \\ 0.0039 & 0.0054 & 0.0039 & 0.0810 \end{bmatrix}.$$

Given this composite density matrix ρ_c, we can now obtain the density matrix representing the state of the decision maker at next time instant in a form of reduced density matrix ρ'_M. This form can be obtained by performing the partial trace of the composite density matrix ρ_c over the environment degrees of freedom, $\rho'_M = \mathrm{Tr}_E\rho_{(M+E)}$, which results in the following normalized evolved probability density matrix of the decision maker after $\Delta t = 1$ time step

$$\rho'_M(t+\Delta t) = \begin{bmatrix} 0.707 & 0.010 \\ 0.010 & 0.292 \end{bmatrix}.$$

VII. CONCLUSION

Open quantum system offers a framework to model and study human decision making and in particular, when it is possible to influence the decision process with some external controlled environment. This controlled environment can take many engineering forms where the feedback to the human can be presented through multiple modalities. The time evolution of the state of the density operator can be described through solution of the master equation. As a part of the solution steps through the Kraus operators, Lindblad operators can be defined which can model the contributions of various other external remodelled dynamics that can contribute to the discrete time evolution of the density operator.

One of the biggest challenges of the above modelling is the definition and interpretation of various terms. For example, Lindblad operators are used to describe the interaction between the system and its environment. Understanding the interpretation and the form of these operators in the context of decision making as a part of both the human and interacting machine is not clear. Also the interplay between the components of Eq. (3) in the context of human-machine interaction remains to be part of the future study.

One possible interpretation of the environmental factors which can be used to influence the decisions of human in performing various tasks can be viewed as assembly of various virtual fixtures. Such interpretation was used as an effective approach in guiding the human user in a virtual environment in performing simple tasks [31, 32]. User in the loop guidance feedback was created through deterministic configuration and feedback. New association of such virtual fixtures to the definition of Lindblad operators can be used to create a library of fixtures, where a collection of them can be activated based on the probability density representing the state of the human decision maker. Machine learning tools can also be utilized in order to create an efficient retrieval of such fixtures as a function of the state of the decision maker along the sequential trajectory.

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