Hedonic Coalition Game Based Distributed Fault-Tolerant Task Allocation for Aircraft Swarm

Yuan Ni and Hao Yang

***Abstract*—This paper investigates the task allocation problem of aircraft swarms. Considering the information of aircraft, targets, no-fly zones, and task constraints, the local optimization performance index of the individual aircraft is designed. Then, a distributed fault-tolerant target allocation approach is proposed based on the hedonic coalition game and the maximum gain message algorithm. With this approach, the task allocation result, which is the Nash equilibrium of the game, can be obtained even under complete failure faults. Simulations further verify the effectiveness of the proposed approach.**

***Index Terms*—Task allocation, aircraft swarm, hedonic coalition game, fault-tolerant control**

I. Introduction

AIRCRAFT swarm is a kind of system composed of a large number of aircraft with different types and simple structures. Through the functional complementation and cooperation between aircraft, it can adapt to complex and diverse task scenarios, such as military operation [1, 2] and environmental monitoring [3, 4], and greatly improve the efficiency and success rate of task execution.

Task allocation is one of the core modules in the decision-making layer of aircraft swarms. As an action guide to carry out tasks, it aims to achieve the matching relationship between aircraft and tasks by analyzing the information of aircraft, task, and environment, and provides an important basis for subsequent flight path planning and individual control of aircraft [5]. Due to the huge scale of the aircraft swarm, traditional centralized task allocation methods, such as mixed integer linear programming [6] and particle swarm algorithm [7], are often inapplicable. Therefore, numerous distributed task allocation methods have emerged, mainly including the market-based method, the distributed constraint optimization method, and the game theory based method.

Among them, the market-based method simulates the mechanism of auction-bid-award in bidding to select appropriate executors for tasks [8, 9]. The distributed constraint optimization method allocates task lists among individuals, which often uses approximation algorithms like max-sum and swarm-gap to deal with the high computational complexity [10–12]. Both methods belong to the top-down methodology. Such a methodology decomposes the complex task into simpler subtasks through hierarchical frameworks, which demand high-performance communication environments.

On the contrary, the game theory based method belongs to the bottom-up methodology. It gives each aircraft the ability of information collection and decision-making, thus realizing task allocation by swarm intelligence. Therefore, the game theory based method is more robust and insensitive to the scale of the swarm. The existing game theory based task allocation methods can be divided into two types. (1) The first type focuses on the task distribution of the swarm from a macro perspective, rather than paying attention to the decision of individuals [13–15]. Concretely, the proportion of aircraft performing various tasks is defined as the state of the swarm, and it can be further modeled and analyzed by evolutionary game theory. (2) The second type designs the decision-making rule of individuals, and then the swarm can converge to the task allocation result through individual strategy adjustment. Such methods usually use the theory of potential game [16], coalition formation game [17, 18], etc.

However, there are some limitations to the above two types of methods. The first type models the state of the whole swarm. Although it makes the final result of task allocation easy to solve, it cannot obtain the decision-making result of each individual. The second type requires that only one individual adjusts its strategy in one time step when solving task allocation results. Therefore, the above two types of methods are both difficult to realize under the distributed architecture in practical engineering.

Moreover, due to the huge scale of the aircraft swarm, complete failure faults of aircraft often occur. It is difficult to directly adjust the targets of each aircraft by operators. However, there is little research on fault-tolerant task allocation. In most cases, the fault is only regarded as a new constraint, and then the tasks are reallocated [19, 20]. This greatly limits the real-time capability and reliability of task allocation.

Thus, it is necessary to study a fully distributed fault-tolerant task allocation method that is easy to realize in engineering. To handle this problem, we propose a distributed

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task allocation approach based on the hedonic coalition game [21] and the maximum gain message algorithm [22], which can ensure the optimization and convergence of task allocation in a fully distributed way, even in the case of complete failure faults. The main contributions are as follows:

(1) Considering the aircraft, the targets, and the no-fly zones, a distributed task allocation model is established based on the hedonic coalition game. The existence of the Nash equilibrium of such a game is proved mathematically.

(2) Based on the established model and the maximum gain message algorithm, a distributed fault-tolerant approach is designed to seek the Nash equilibrium. The convergence of the algorithm is proved mathematically.

The rest of the paper is as follows. In Section II, the optimization problem of the task allocation for the aircraft swarm is established. In Section III, a distributed fault-tolerant task allocation approach is designed based on the hedonic coalition game and the maximum gain message algorithm. Simulations are presented in Section IV, followed by the conclusion in Section V.

II. Problem Formulation

A. Task Allocation Scenario

The task allocation scenario considered in this paper is shown in Fig. 1, including an aircraft swarm with $m>0$ members, $0<p<m$ targets, and $q>0$ no-fly zones. Denote the set of aircraft as $M \overset{\Delta}{=} \{1,2,\dots,m\}$, and the coordinate of aircraft $i \in M$ as (x_i^M, y_i^M). Denote the set of targets as $\bar{P} = P \cup \{0\}$, where the set $P \overset{\Delta}{=} \{1,2,\dots,p\}$ corresponds to the real targets and the target $\{0\}$ corresponds to standby. The coordinate of target $j \in P$ is (x_j^P, y_j^P), and its value is V_j, i.e., the income obtained after the target is successfully struck. Especially, the value of standby is 0. Denote the set of no-fly zones as $Q \overset{\Delta}{=} \{1,2,\dots,q\}$. The center coordinate of the no-fly zone $l \in Q$ is (x_l^Q, y_l^Q), and its coverage radius is R_l. Note that neither the aircraft nor the target is in the no-fly zone.

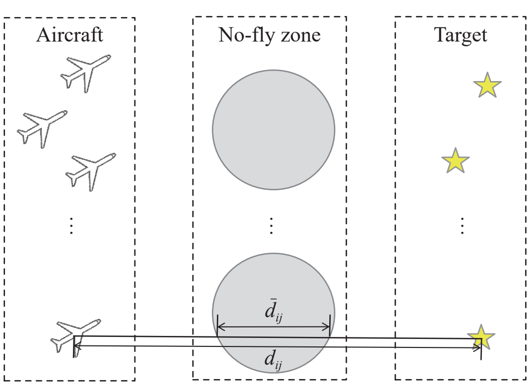

Figure 1 Task allocation scenario.

On this basis, a decision variable matrix $A \overset{\Delta}{=} [\alpha_{ij}]^{m\times(p+1)}$ is defined to characterize the matching relationship between the target and the aircraft. α_{ij} denotes the matching relationship between the aircraft i and the target j. If $\alpha_{ij}=1$, the matching between them is successful. Otherwise, the matching fails.

Inspired by the concept of "single-task robot" in Ref. [23], and considering the particularity of the target strike, it is assumed that each aircraft can only strike one target. Thus, the following target conflict constraint is established.

$$\sum_{i\in M, j\in \bar{P}} \alpha_{ij} = 1 \tag{1}$$

Moreover, the strike range and maneuverability of the aircraft are limited. Therefore, as shown in Fig. 1, when selecting target, it is necessary to consider not only the distance from the aircraft to the target, but also whether the aircraft can avoid the no-fly zones. The distance from aircraft $i \in M$ to target $j \in P$ is denoted as

$$d_{ij} = \sqrt{(x_i^M - x_j^P)^2 + (y_i^M - y_j^P)^2} \tag{2}$$

Denote the distance between the no-fly zones and the connecting line of the aircraft i and the target j as

$$\bar{d}_{ij} = \sum_{l\in Q_{ij}} 2\sqrt{{R_l}^2 - {d_{ijl}}^2} \tag{3}$$

$$\begin{aligned} d_{ijl} = |&(y_i^M - y_j^P)x_l^Q + (x_j^P - x_i^M)y_l^Q + \\ &x_i^M y_j^P - x_j^P y_i^M| d_{ij}^{-1} \end{aligned} \tag{4}$$

where $Q_{ij} = \{l \in Q | d_{ijl} < R_l\}$ is the no-fly zone set that the aircraft i needs to avoid to reach the target j.

For aircraft $i \in M$ and target $j \in P$, the following target matching constraint is established.

$$\begin{cases} \alpha_{ij} \in \{0,1\},\, D_{ij} \leqslant \lambda \text{ or } j=0; \\ \alpha_{ij} = 0,\, D_{ij} > \lambda \end{cases} \tag{5}$$

where λ is a positive constant, and

$$D_{ij} = \rho_i d_{ij} + \bar{\rho}_i \bar{d}_{ij} \tag{6}$$

where $\rho_i > 0$ is a parameter that represents the long-range strike capability of aircraft i, and $\bar{\rho}_i$ is a parameter to characterize the maneuverability of aircraft i.

B. Communication Topology

An undirected graph $G \overset{\Delta}{=} (P, \mathcal{E})$ is established to represent the communication topology of the aircraft swarm. The node set corresponds to the real target set P, and the edge set $\mathcal{E} \subseteq P \times P$ corresponds to the set of communication links. An edge $(i,k) \in \mathcal{E}$ means that aircraft i and k can communicate with each other. Denote $N_i \overset{\Delta}{=} \{k \in P | (i,k) \in \mathcal{E}\}$ as the neighbour set of aircraft i. The adjacency matrix of the undirected graph G is defined as $B \overset{\Delta}{=} [\beta_{ik}]^{m\times m}$, where if $(i,k) \in \mathcal{E}$, $\beta_{ik} = \beta_{ki} = 1$, otherwise $\beta_{ik} = \beta_{ki} = 0$.

According to the target matching constraint in Eq. (5), the real target set that the aircraft $i \in M$ can select can be defined as $P_i \overset{\Delta}{=} \{j \in P | D_{ij} \leqslant \lambda\}$. To save communication resource and reduce communication interference, we make Assumption 1.

Assumption 1 For any two aircraft, they communicate with each other if and only if they can choose the same real target, i.e., for $i, k \in M$, if $P_i \cap P_k \neq \emptyset$ and $i \neq k$, then $\beta_{ik} = \beta_{ki} = 1$, otherwise $\beta_{ik} = \beta_{ki} = 0$.

Moreover, to ensure the interaction between aircraft, Assumption 2 is given.

Assumption 2 For $i \in M$, the neighbour set of aircraft i is not an empty set, i.e., $N_i \neq \emptyset$.

C. Optimization Problem of Task Allocation

The strike efficiency of aircraft $i \in M$ against the target $j \in P_i$ is related to distance d_{ij} and distance $\bar{d}_{ij}$. The smaller d_{ij} and $\bar{d}_{ij}$ are, the greater the strike efficiency is. According to this relationship, the strike efficiency function of aircraft i to target j, that is, the probability function of aircraft i successfully striking target j, is established as

$$u_{ij} = \mathrm{e}^{-D_{ij}} \tag{7}$$

Because the number of targets, p, is smaller than that of aircraft, m, there is often a situation in which multiple aircraft cooperate to strike the same target. The set of aircraft that currently select target $j \in \bar{P}$ is defined as Λ_j. According to the attack efficiency function in Eq. (7), for $j \in P$, the probability that target j is successfully struck can be expressed as

$$u_j = 1 - \prod_{i \in \Lambda_j} (1 - u_{ij}) \tag{8}$$

Especially, for $j = 0$, the probability that target j is successfully struck is set as 1.

If the aircraft set Λ_j successfully strikes target j, then the value V_j is obtained by the cooperation of multiple aircraft and needs to be allocated to each aircraft in set Λ_j. Therefore, if aircraft i chooses target j, its utility function can be expressed as

$$r_{ij}(j, |\Lambda_j|) = \frac{V_j u_j}{|\Lambda_j|} = \frac{V_j \left(1 - \prod_{i \in \Lambda_j} (1 - \mathrm{e}^{-D_{ij}})\right)}{|\Lambda_j|} \tag{9}$$

where $|\Lambda_j|$ represents the number of aircraft in set Λ_j.

According to the communication topology of aircraft swarm, aircraft $i \in M$ can only receive the information of the aircraft in its neighbour set N_i. Therefore, the local optimization objective of aircraft i is established as

$$\max_{s_i \in P_i \cup \{0\}} J_i(s_i, s_{N_i}) \tag{10}$$

$$J_i(s_i, s_{N_i}) = \sum_{k \in N_i \cup \{i\}, s_k \in P_k \cup \{0\}} \alpha_{ks_k} r_{ks_k}(s_k, |\Lambda_{s_k}|) \tag{11}$$

where $J_i(s_i, s_{N_i})$ is the local optimization performance index of aircraft i, s_i is the target of aircraft i, and s_{N_i} is the target set of neighbourhood of aircraft i. It can be seen that the local optimization goal of aircraft i is to maximize the total utility of the aircraft itself and its neighbours.

On the basis of the aircraft set M, the target set $\bar{P}$, and the performance index J_i, a game framework is formed. Before presenting the research goal of this paper, we give the definition of Nash equilibrium under the game framework.

Definition 1 The set of targets $\{s_1^*, s_2^*, ..., s_m^*\}$ is a Nash equilibrium solution if all $i \in M$

$$J_i^* = J_i(s_i^*, s_{-i}^*) \geqslant J_i(s_i, s_{-i}^*) \tag{12}$$

where $s_{-i}^* \overset{\Delta}{=} \{s_k | k \neq i, k \in M\}$ denotes the set of the optimal targets of all aircraft except i. The N-tuple of the local optimization performance indices $\{J_1^*, J_2^*, ..., J_m^*\}$ is known as the Nash equilibrium.

Therefore, the research goal of this paper is to design a distributed approach for each aircraft to realize the local optimization objective in Eq. (10) and to make the task allocation of the aircraft swarm converge to the Nash equilibrium, even if it encounters complete failure faults.

III. HEDONIC COALITION GAME BASED DISTRIBUTED APPROACH

A. Hedonic Coalition Game Model

Hedonic coalition game is a kind of cooperative game, where players are partitioned into several coalitions according to their own preferences during the game and finally form a Nash stable coalition partition. Given the set of players as $M_\mathrm{p} = \{1, 2, \ldots, m_\mathrm{p}\}$ and the set of strategies as $P_\mathrm{s} = \{1, 2, \ldots, p_\mathrm{s}\}$, the definitions of coalition partition, preference relation, and Nash stable coalition partition are detailed in Definitions 2–4.

Definition 2 For the strategy set P_s, coalition partition of set M_p is defined as $\Omega_\mathrm{c} \overset{\Delta}{=} \{\Lambda_{\mathrm{c}_1}, \Lambda_{\mathrm{c}_2}, \ldots, \Lambda_{\mathrm{c}_p}\}$, where for $j_\mathrm{s} \in P_\mathrm{s}$, $\Lambda_{\mathrm{c}_j} \subseteq P_\mathrm{s}$ represents the player set that selects the strategy j_s, called coalition Λ_{c_j}. It satisfies that $\bigcup_{j_\mathrm{s} \in P_\mathrm{s}} \Lambda_{\mathrm{c}_j} = M_\mathrm{p}$.

Definition 3 For $i_\mathrm{p} \in M_\mathrm{p}$, $\succ_{i_\mathrm{p}}$ represents the preference relation of the player to the coalitions. For any two coalitions Λ_{c_j}, $\Lambda_{\mathrm{c}_\mu} \in \Omega_\mathrm{c}$, if $\Lambda_{\mathrm{c}_j} \succ_{i_\mathrm{p}} \Lambda_{\mathrm{c}_\mu}$, it means that the player i_p prefers to join coalition Λ_{c_j} rather than join coalition Λ_{c_μ}.

Definition 4 A coalition partition Ω_c is Nash stable if each player prefers its current coalition to other coalitions. Such a coalition partition can be represented by $\Omega_\mathrm{c}^* = \{\Lambda_{\mathrm{c}_1}^*, \Lambda_{\mathrm{c}_2}^*, \ldots, \Lambda_{\mathrm{c}_p}^*\}$.

According to the target conflict constraint in Eq. (1), task allocation of the aircraft swarm M to the targets $\bar{P}$ is consistent with the definition of coalition partition. Therefore, the set of aircraft that select target $j \in \bar{P}$ can be redefined as coalition Λ_j, and the coalition partition of swarm can be defined as $\Omega \overset{\Delta}{=} \{\Lambda_0, \Lambda_1, \ldots, \Lambda_p\}$.

Based on Eq. (9), the utility of the coalition Λ_j that matches target j can be denoted as

$$R(j, |\Lambda_j|) = \sum_{i \in \Lambda_j} \alpha_{ij} r_{ij}(j, |\Lambda_j|) \tag{13}$$

Moreover, according to the local optimization objective in Eq. (10), to make the aircraft gain more utility when choosing the coalition, the preference relation of aircraft $i \in M$ is designed as Eq. (14).

For coalitions Λ_j, $\Lambda_\mu \in \Omega$, where j, $\mu \in P_i \cup \{0\}$, if

$$\begin{aligned} &R(j, |\Lambda_j \cup \{i\}|) + R(\mu, |\Lambda_\mu \backslash \{i\}|) > \\ &R(j, |\Lambda_j \backslash \{i\}|) + (\mu, |\Lambda_\mu \cup \{i\}|) \end{aligned} \tag{14}$$

then $\Lambda_j \succ_i \Lambda_\mu$. Based on the preference relation, the target selection rule of aircraft is as follows.

Given a coalition partition $\Omega = \{\Lambda_0, \Lambda_1, \ldots, \Lambda_p\}$, for any aircraft $i \in M$, if and only if there is a target $\bar{s}_i \in P_i \cup \{0\}$,

$\bar{s}_i \neq s_i$ which satisfies $\Lambda_{\bar{s}_i} \succ_i \Lambda_{s_i}$, then the aircraft i will adjust the currently matched target s_i to target $\bar{s}_i$, that is, aircraft i will move from coalition Λ_{s_i} to coalition $\Lambda_{\bar{s}_i}$. Accordingly, the coalition partition will change from Ω to $\{\Omega \backslash \{\Lambda_{s_i}, \Lambda_{\bar{s}_i}\}\} \cup \{\Lambda_{s_i} \backslash \{i\}, \Lambda_{\bar{s}_i} \cup \{i\}\}$.

Remark 1 As can be seen from Eq. (14), when choosing a new coalition, aircraft i considers not only its own utility, but also the total utilities of the coalitions it leaves and joins. This not only reflects the idea of cooperation, but also ensures that its performance index in Eq. (11) will rise when it changes coalitions.

According to the aircraft set M, the target set $\bar{P}$, and the target selection rule based on the preference relation, a hedonic coalition game model is established. Denote the Nash stable coalition partition of the game as $\Omega^* = \{\Lambda_0^*, \Lambda_1^*, \ldots, \Lambda_p^*\}$. Under this Nash stable coalition partition, for any aircraft $i \in M$ and target $s_i \in P_i \cup \{0\}$, $s_i \neq s_i^*$, it satisfies $\Lambda_{s_i^*} \succ_i \Lambda_{s_i}$. Each aircraft prefers the currently selected target, and none of them can improve its performance index by changing its target.

Theorem 1 Under the Nash stable coalition partition Ω^*, the set of performance indices of the aircraft swarm, i.e., $\{J_1^*, J_2^*, \ldots, J_m^*\}$, constitutes a Nash equilibrium.

The proof of Theorem 1 is presented in Appendix A.

Thus, the proposed hedonic coalition game model can be applied to seek the Nash equilibrium of Definition 1.

B. Distributed Algorithm for Seeking the Nash Equilibrium

Before seeking the Nash equilibrium, we first prove its existence using the potential game theory. Given the player set M_p, the definition of potential game is given in Definition 5.

Definition 5 A game is a class of potential game if for any $i_\mathrm{p} \in M_\mathrm{p}$ there exsits a potential function $S \to \Phi$ that satisfies

$$\Phi(s_{i_\mathrm{p}}, s_{-i_\mathrm{p}}) - \Phi(s'_{i_\mathrm{p}}, s_{-i_\mathrm{p}}) = J_{i_\mathrm{p}}(s_{i_\mathrm{p}}, s_{-i_\mathrm{p}}) - J_{i_\mathrm{p}}(s'_{i_\mathrm{p}}, s_{-i_\mathrm{p}}) \quad (15)$$

where S refers to the strategy space of all players, s_{i_p} and s'_{i_p} stand for strategies of player i_p, s_{-i_p} represents the strategy set of other players except player i_p, and J_{i_p} is the performance index of player i_p, respectively.

Moreover, if the players and strategy space of the potential game are limited, it is a finite potential game.

Based on Ref. [24], the finite potential game has the following two properties:

(1) There must be at least one pure strategy Nash equilibrium.

(2) The improvement steps of the potential function are limited.

Next, we prove the existence of Nash equilibrium by using the properties of potential game.

Theorem 2 There exists at least one Nash equilibrium for the hedonic coalition game modeled by the aircraft set M, the target set $\bar{P}$, and the target selection rule.

The proof of Theorem 2 is presented in Appendix B.

To make the aircraft swarm reach the Nash equilibrium through local information interaction under the communication topology, a distributed algorithm for aircraft $i \in M$ in one iteration is established. Flow chart of this algorithm is shown in Fig. 2.

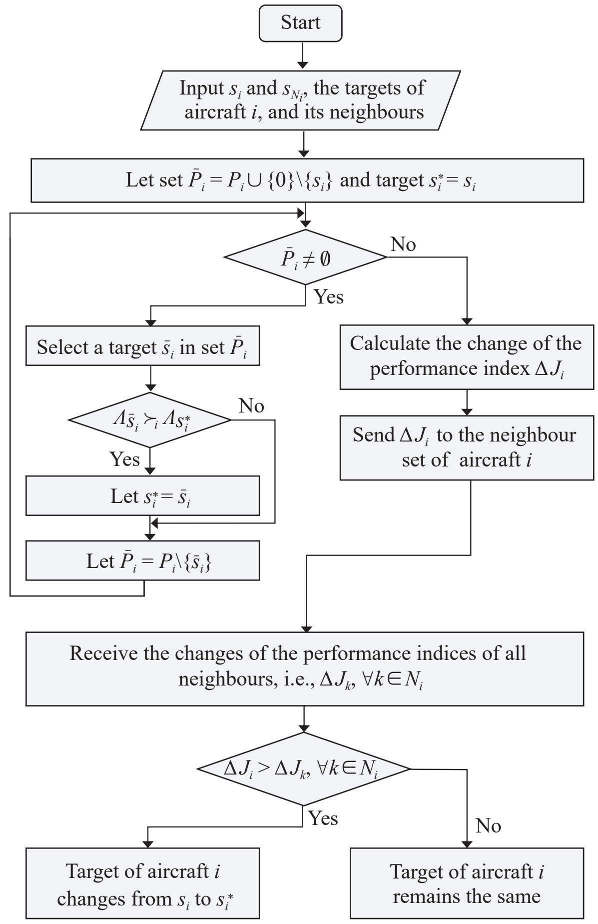

Figure 2 Flow chart of the distributed task allocation algorithm.

The distributed task allocation algorithm consists of two parts.

Part I: Target selection and information transmission. Part I is based on the preference relation in Eq. (14) and the target selection rule of aircraft.

Step 1 Introduce s_i^* to store the target selected by aircraft i, and its initial value is s_i. Introduce a set $\bar{P}_i$ to avoid repeated selection of the same target, and its initial value is set as $P_i \cup \{0\} \backslash \{s_i\}$.

Step 2 Judge whether set $\bar{P}_i$ is an empty set, if it is, skip to step 3. If it is not, select any target $\bar{s}_i$ in set $\bar{P}_i$. If it is satisfied that $\Lambda_{\bar{s}_i} \succ_i \Lambda_{s_i^*}$, it means that the aircraft prefers to join coalition $\Lambda_{\bar{s}_i}$ rather than join coalition $\Lambda_{s_i^*}$. Thus, we assign $\bar{s}_i$ to s_i^*, remove $\bar{s}_i$ from set $\bar{P}_i$, and skip to step 2.

Step 3 The increase of the optimization performance index when the aircraft i moves from coalition $\Lambda_{s_i^*}$ to coalition $\Lambda_{\bar{s}_i}$ is defined as

$$\Delta J_i = J_i(\bar{s}_i, s_{N_i}^*) - J_i(s_i^*, s_{N_i}^*) \quad (16)$$

Send ΔJ_i to the neighbourhood of aircraft i. Especially, if $s_i^* = s_i$ and $\Delta J_i = 0$.

Part II: Information reception and target update. Part II is based on the idea of the maximum gain message algorithm [22].

According to the received information about the changes of the performance indices of the aircraft in the neighbourhood N_i, judge whether it is satisfied that $\Delta J_i > \Delta J_k, \forall k \in N_i$. If it is satisfied, it means that the increase of the performance index of aircraft i is greater than that of its neighbours, then aircraft i changes its target from s_i to s_i^*. If it is not satisfied, it means that there is a neighbour whose performance index increase is greater than or equal to that of aircraft i. Then, after this iteration, the aircraft still chooses target s_i.

Theorem 3 By applying the proposed distributed task allocation algorithm, the aircraft swarm can converge to the Nash equilibrium.

The proof of Theorem 3 is presented in Appendix C.

Next, we analyze the computational complexity of the algorithm.

For the time complexity, in one iteration, aircraft i needs to traverse the target set $\bar{P}_i$ and compare its increase of performance index with all neighbors in N_i. Because this distributed algorithm is executed by each aircraft in parallel, the time complexity of one iteration is $O(n_{\max})$, where $n_{\max} = \max_{i\in M}(|\bar{P}_i| + |N_i|)$. Suppose the number of iterations is T, the time complexity of the algorithm is $O(Tn_{\max})$.

For the space complexity, considering that the aircraft needs to store the target set $\bar{P}_i$ and the increases of performance indices $\Delta J_k, \forall k \in N_i$, the space complexity of aircraft i is $O(|\bar{P}_i| + |N_i|)$. Therefore, the space complexity of the aircraft swarm is $O(n_{\text{sum}})$, where $n_{\text{sum}} = \sum_{i\in M}(|\bar{P}_i| + |N_i|)$.

C. Fault-Tolerant Characteristic of the Algorithm

The proposed distributed task allocation algorithm can tolerate complete failure faults. As can be seen from Fig. 2, if for any healthy aircraft, its currently selected target is not the optimal target that maximizes its performance index after removing the failed aircraft, then at least one of the aircraft and its neighbours will change the target in this iteration. After that, the swarm will continue to iterate until the target selected by each aircraft is the optimal target to maximize its own performance index.

Remark 2 In addition to tolerating complete failure faults, the proposed distributed task allocation algorithm can also adapt well to the situation of the increase of aircraft. Therefore, it is easy to see that this algorithm is insensitive to the scale of aircraft swarms and has great application potential for large-scale swarms.

IV. Simulation

A. Fault-Free Case

The task allocation scenario consisting of 10 aircraft, 4 targets, and 1 no-fly zone is considered. Tables 1–3 show the coordinate of each aircraft, the coordinate and value of each target, and the center coordinate and coverage radius of each no-fly zone, respectively.

For $i \in M$, let $\rho_i = \rho = 0.016$, $\bar{\rho}_i = \bar{\rho} = 0.010$, and $\lambda = 1$. According to the target matching constraint, the real target set that each aircraft can select is

$$
\begin{aligned}
&P_1 = P_2 = P_3 = \{1,2\},\\
&P_4 = P_5 = P_6 = \{2,3\},\\
&P_7 = P_8 = P_9 = P_{10} = \{3,4\}
\end{aligned} \tag{17}
$$

Thus, according to Assumption 1, the communication topology of the aircraft swarm is

$$
B = \begin{bmatrix} 1^{3\times3} & 1^{3\times3} & 0^{3\times4} \\ 1^{3\times3} & 1^{3\times3} & 1^{3\times4} \\ 0^{4\times3} & 1^{4\times3} & 1^{4\times4} \end{bmatrix} - 1^{10\times10} \tag{18}
$$

where $1^{m\times n}$ and $0^{m\times n}$ are matrices of one and zero, respectively.

Table 1 Coordinate of aircraft.

Aircraft number	x-axis coordinate	y-axis coordinate
1	15	12
2	5	20
3	2	30
4	6	45
5	8	50
6	9	56
7	8	63
8	13	70
9	14	82
10	16	86

Table 2 Coordinate and value of target.

Target number	x-axis coordinate	y-axis coordinate	Value
1	60	3	6
2	55	19	4
3	67	58	4
4	70	72	3

Table 3 Information of no-fly zone.

No-fly zone number	x-axis coordinate	y-axis coordinate	Radius
1	40	33	6

At the initial time, all aircraft stand by. The task allocation result is shown in Fig. 3.

As can be seen from Fig. 3, the coalitions of each target are

$$
\begin{aligned}
&\Lambda_0 = \emptyset,\\
&\Lambda_1 = \{1,2,3\},\\
&\Lambda_2 = \{4,5\},\\
&\Lambda_3 = \{7,8,9\},\\
&\Lambda_4 = \{6,10\}
\end{aligned} \tag{19}
$$

Figure 4 shows the trajectory of the total utility. It can be seen that under the proposed distributed task allocation algorithm, the total utility strictly increases and finally converges in the 10th iteration.

B. Faulty Case

In order to compare with the fault-free case, the coordinate

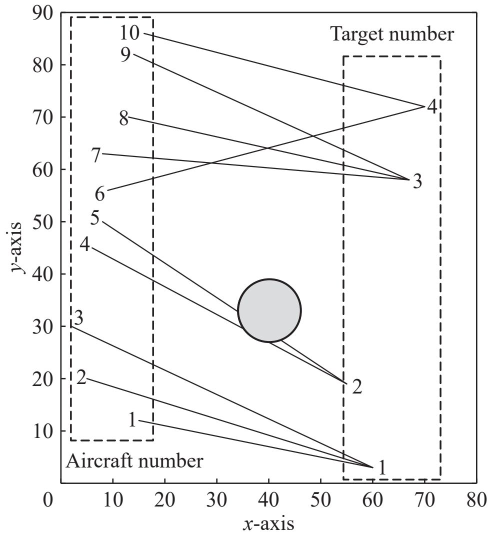

Figure 3 Task allocation result in the fault-free case.

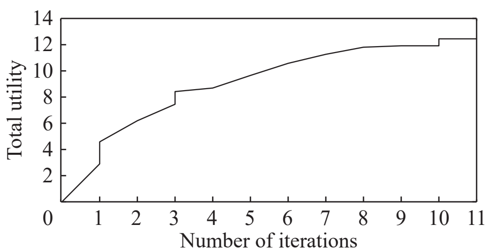

Figure 4 Trajectory of the total utility in the fault-free case.

of each aircraft, the coordinate and value of each target, and the center coordinate and the coverage radius of each no-fly zone are the same as those in Tables 1–3. The progress of the task allocation is based on the result in the fault-free case. The aircraft set with complete failure fault is considered as $\{6,10\}$. The task allocation result of the swarm in the faulty case is shown in Fig. 5.

As can be seen from Fig. 5, the coalitions of each target are

$$\begin{aligned} \Lambda_0 &= \emptyset, \\ \Lambda_1 &= \{1,2,3\}, \\ \Lambda_2 &= \{4,5\}, \\ \Lambda_3 &= \{7,8\}, \\ \Lambda_4 &= \{9\} \end{aligned} \tag{20}$$

Figure 6 shows the trajectory of the total utility. It can be seen that the total utility of the swarm decreases due to the occurrence of the complete failure fault in the 11th iteration. Then, under the proposed distributed task allocation algorithm, the total utility of the swarm increases and finally converges in the 13th iteration.

V. Conclusion

In this paper, we propose a distributed fault-tolerant task allocation approach for aircraft swarms based on the hedonic coalition game and the maximum gain message algorithm. The existence of the Nash equilibrium and its convergence are both proved mathematically. Simulation results show that the proposed approach can optimize task allocation and ensure convergence, even in the presence of faults. It indicates that this research provides a practical and robust method for task allocation of large-scale aircraft swarms. However, this paper only pays attention to the decision-making level, and we will further study the joint design of the decision-making layer and control layer in the future.

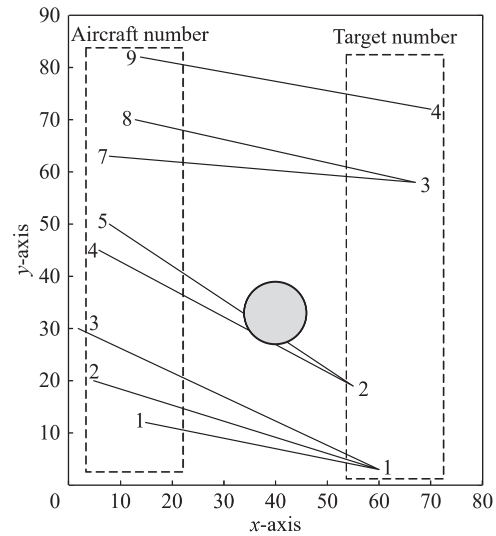

Figure 5 Task allocation result in the faulty case.

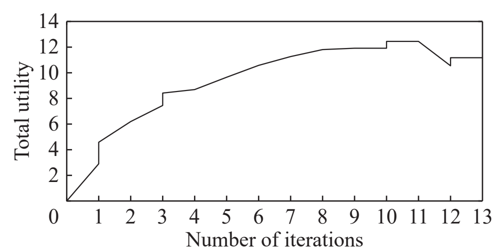

Figure 6 Trajectory of the total utility in the faulty case.

Appendix A
Proof of Theorem 1

Proof Assuming the set of performance indices of the aircraft swarm, i.e., $\{J_1^*, J_2^*, ..., J_m^*\}$, is not a Nash equilibrium. Thus, for $i \in M$, there exists a target $\hat{s}_i \neq s_i^*$ satisfying

$$J_i(\hat{s}_i, s_{-i}^*) > J_i(s_i^*, s_{-i}^*) \tag{A1}$$

which implies that

$$\begin{aligned} &R(\hat{s}_i, |\Lambda_{\hat{s}_i} \cup \{i\}|) + R(s_i^*, |\Lambda_{s_i^*} \backslash \{i\}|) > \\ &R(\hat{s}_i, |\Lambda_{\hat{s}_i} \backslash \{i\}|) + R(s_i^*, |\Lambda_{s_i^*} \cup \{i\}|) \end{aligned} \tag{A2}$$

It follows that $\Lambda_{\hat{s}_i} \succ_i \Lambda_{s_i^*}$, which contradicts that Ω^* is a Nash stable partition. This completes the proof. ■

Appendix B
Proof of Theorem 2

Proof Define the potential function as

$$\Phi = \sum_{i\in M, s_i\in P_i\cup\{0\}} \alpha_{is_i} r_{is_i}(s_i, |\Lambda_{s_i}|) \tag{A3}$$

Thus, we have

$$\begin{aligned}&\Phi(s_i, s_{-i}) - \Phi(s_i', s_{-i}) = \\ &\quad R(s_i, |\Lambda_{s_i}\cup\{i\}|) + R(s_i', |\Lambda_{s_i'}\backslash\{i\}|) - \\ &\quad R(s_i, |\Lambda_{s_i}\backslash\{i\}|) - R(s_i', |\Lambda_{s_i'}\cup\{i\}|) = \\ &\quad J_i(s_i, s_{N_i}) - J_i(s_i', s_{N_i}) = \\ &\quad J_i(s_i, s_{-i}) - J_i(s_i', s_{-i})\end{aligned} \tag{A4}$$

With reference to Definition 5, it follows that the proposed hedonic coalition game in Section III is a class of potential game. Moreover, according to the definition of the aircraft set M and the target set P, the game is further a finite potential game. Therefore, according to the first property of finite potential games, there must exist a Nash equilibrium. This completes the proof. ■

APPENDIX C
PROOF OF THEOREM 3

Proof Denote $M^{(\tau)}$ as the set of aircraft that change their targets in the τ-th iteration. It is assumed that $M^{(\tau)} \neq \emptyset$, otherwise the aircraft swarm has already converged to the Nash equilibrium. If $i \in M^{(\tau)}$, then according to the proposed algorithm, the aircraft in N_i cannot change their targets in the τ-th iteration. Based on Assumption 2, it follows that $M\backslash M^{(\tau)} \neq \emptyset$.

Thus, for $i \in M^{(\tau)}$, we have

$$r_{is_i}^{(\tau+1)}(s_i, |\Lambda_{s_i}|) > r_{is_i}^{(\tau)}(s_i, |\Lambda_{s_i}|) \tag{A5}$$

for $i \in M\backslash M^{(\tau)}$, we have

$$r_{is_i}^{(\tau+1)}(s_i, |\Lambda_{s_i}|) = r_{is_i}^{(\tau)}(s_i, |\Lambda_{s_i}|) \tag{A6}$$

where $r_{is_i}^{(\tau+1)}(s_i, |\Lambda_{s_i}|)$ and $r_{is_i}^{(\tau)}(s_i, |\Lambda_{s_i}|)$ correspond to the utilities of aircraft k in the τ-th iteration and the $(\tau+1)$-th iteration, respectively.

We use the potential function Φ to analyze the change of the global utility in the τ-th iteration.

$$\begin{aligned}&\Phi^{(\tau+1)} = \sum_{i\in M} \alpha_{is_i} r_{is_i}^{(\tau+1)}(s_i, |\Lambda_{s_i}|) = \\ &\quad \sum_{i\in M^{(\tau)}} \alpha_{is_i} r_{is_i}^{(\tau+1)}(s_i, |\Lambda_{s_i}|) + \\ &\quad \sum_{i\in M\backslash M^{(\tau)}} \alpha_{is_i} r_{is_i}^{(\tau+1)}(s_i, |\Lambda_{s_i}|) > \\ &\quad \sum_{i\in M^{(\tau)}} \alpha_{is_i} r_{is_i}^{(\tau)}(s_i, |\Lambda_{s_i}|) + \\ &\quad \sum_{i\in M\backslash M^{(\tau)}} \alpha_{is_i} r_{is_i}^{(\tau)}(s_i, |\Lambda_{s_i}|) > \\ &\quad \Phi^{(\tau)}\end{aligned} \tag{A7}$$

It can be seen from Eq. (A7) that the global utility of the aircraft swarm is strictly increasing. According to the proof of Theorem 1, the proposed hedonic coalition game in Section III is a finite potential game. Based on the second property of the finite potential game, the increase steps of Φ are limited, so by applying the proposed distributed algorithm, the aircraft swarm can converge to the Nash equilibrium. This completes the proof. ■

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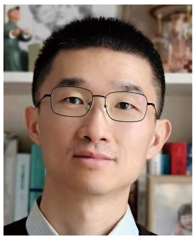

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