

Distributed Control Strategy for Rotor Speed Synchronization in Multiple PMSMs Based on Port-Hamiltonian Dynamics

Jingyi Zhao, Yongxin Wu, Zhenhua Zhang, and Yuhu Wu

Abstract—In this paper, the synchronous rotor speed control problem of multiple permanent magnet synchronous motors (PMSMs) with port-Hamiltonian (PH) dynamics is investigated. First, the synchronous rotor speed problem is reformulated as an optimization problem and a distributed controller is proposed for each PMSM. Then, the stability of the closed-loop system is analyzed, and it is proven that its equilibrium aligns with the optimum of the value function derived from the optimization problem. Based on the proposed distributed controller, PMSMs exchange the estimated average rotor speed with their neighbours, rather than the direct rotor speed, which benefits the protection of the sensitive state variables. Finally, a simulation example is presented to validate the effectiveness of the proposed distributed controller.

Index Terms—Port-Hamiltonian (PH) system, distributed control, permanent magnet synchronous motor (PMSM), rotor speed synchronization

I. INTRODUCTION

PERMANENT magnet synchronous motors (PMSMs), including both surface-mounted and internal permanent magnet types, exhibit distinctively preponderant characteristics in the field of variable speed drives. These remarkable traits include high power density, favorable dynamics, exceptional efficiency, and a wide operating speed range, as shown in Refs. [1, 2]. Consequently, PMSMs are widely used in various industrial applications, including servo drives, high speed trains, electric vehicles, and household appliances, as indicated in Ref. [3].

However, in certain practical engineering applications, a single PMSM may not suffice to meet the required demands. In such cases, it becomes necessary for multiple PMSMs to operate in a coordinated manner to fulfill the engineering requirements. For example, in the manufacturing-oriented market environment, driven by the pursuit of fabricating

commodities of superior manufacturing quality at a diminished production related cost, the manufacturing sector has proposed manufacturing-operational strategies centring on synchronizing two or more rotational shafts within mechanical assemblies. However, these established approaches are encumbered with several drawbacks [4], including prolonged part-replacement time, uncertainties because of wear-related degradation, imprecise speed trajectories, and relatively low mechanical-transmission reliability. To overcome these challenges, rotor speed synchronization control of multiple motors has emerged as a highly effective solution [5]. Consequently, the rotor speed synchronization control of multiple motors is highly significant as it allows each shaft, driven and controlled by an individual motor without physical connection to others, to maintain synchronized operation during acceleration, deceleration, and varying load conditions. This ensures the stability and consistency of the manufacturing process. As demonstrated in Ref. [6], implementing multi-motor rotor speed synchronization control leads to significant improvements in production output, product quality, and energy efficiency.

There are some researches related to the speed synchronous control of multiple motors. For example, Zhao et al. [7] investigated a real-time speed synchronous control approach for multiple induction motors with speed acceleration and load changing based on the sliding mode control theory. Wang et al. [8] proposed a novel tracking and synchronization control strategy for multi-motor driving systems with unknown parameters including two subcontrollers: the adaptive immersion and invariance tracking controller and the robust integral of the sign of the error synchronization controller. Hu et al. [9] put forward a robust adaptive synchronization and tracking control strategy which was based on neural network for multi-motor driving servo systems. More details can be found in Ref. [10].

Since the concept of cyber-physical systems was introduced, research in this field has progressed rapidly with growing attention from scholars toward distributed strategies combined with physical systems. In fact, systems in multi-physical domains can be modeled using port-Hamiltonian (PH) frameworks, which offer a broader range of applications compared with the Eule-Lagrange (EL) systems. PH systems inherently benefit from their interconnected structure, making them particularly well-suited for networking [11, 12]. Because

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of its powerful functions, several studies have investigated PH system modeling for PMSM and their control design. Petrovic et al. [13] developed an energy-shaping control method for the speed regulation of PMSMs. Yaghmaei and Yazdanpanah [14] designed a full-order observer using the notion of contractive PH systems and applied it to PMSMs. However, these works focus on the control of a single PMSM rather than the rotor speed synchronous control of multiple PMSMs.

This paper aims to design the distributed controller for each PMSM to achieve the rotor speed synchronous of multiple PMSMs described by PH framework. The main contributions are summarized as follows:

(1) In the field of networked communication control for multiple agents, the PH framework offers distinct advantages over alternative approaches, because of its inherent interconnectivity and extensive application scope. Therefore, we present the design of a distributed rotor speed synchronous controller for multiple PMSMs with PH dynamics. Under the action of the designed controller, the closed-loop system exponentially converges to the equilibrium where all PMSMs achieve rotor speed synchronization.

(2) The proposed distributed controller enables PMSMs to exchange information through estimated values and auxiliary variables, rather than relying on exact state values. This approach effectively reduces the risk of sensitive data disclosure. At the same time, compared with the centralized controller, the proposed distributed controller significantly alleviates the network communication burden.

The paper is organized as follows. In Section II, some preliminaries are introduced. In Section III, the problem is formulated. Then, a distributed controller is proposed in Section IV and the convergence is analyzed. Section V gives an example to verify the proposed algorithm. Finally, Section VI concludes this paper.

Notation \mathbb{R}^n denotes the n -dimensional Euclidean space. $\mathbf{1}_n \in \mathbb{R}^n$ and $\mathbf{0}_n \in \mathbb{R}^n$ represent the vectors of all ones and all zeros, respectively. x^\top is the transpose of x . The set $\{i, i+1, \dots, j-1, j\}$ is described by $[i:j]$, where $i, j \in \mathbb{R}$ and $i < j$. $\text{col}(x_1, x_2, \dots, x_N) = (x_1^\top, x_2^\top, \dots, x_N^\top)^\top \in \mathbb{R}^{Nn}$ with $x_i \in \mathbb{R}^n$, $i \in [1:N]$. I_n is the identity matrix in $\mathbb{R}^{n \times n}$. $\text{diag}(\lambda_1, \lambda_2, \dots, \lambda_n)$ is the diagonal matrix of elements $\lambda_1, \lambda_2, \dots, \lambda_n \in \mathbb{R}$. For a function $f(x, y): \mathbb{R}^m \times \mathbb{R}^n \rightarrow \mathbb{R}$, the partial derivative with respect to x is $\nabla_x f(x, y) = \frac{\partial f(x, y)}{\partial x}$ and the gradient of $f(x, y)$ is $\nabla f(x, y) = \text{col}\left(\frac{\partial f(x, y)}{\partial x}, \frac{\partial f(x, y)}{\partial y}\right)$. The symbol $\ker(L)$ represents the zero space of L while $\text{range}(L)$ represents the range space of L .

II. PRELIMINARY

This section mainly introduces some basic preliminary knowledge used and describes the problem investigated in this work.

A. Basic Concept

Consider an undirected graph $\mathcal{G} := \{\mathcal{V}, \mathcal{E}, \mathcal{A}\}$, where $\mathcal{V} = [1:N]$ and \mathcal{E} denote the node set and the edge set, respectively, and $\mathcal{A} := (a_{ij})_{N \times N}$ denotes the adjacency matrix.

If i is a neighbor of j , then the pair $(i, j) \in \mathcal{E}$ is an edge of \mathcal{G} and $a_{ij} = 1$. Moreover, for all $i \in \mathcal{V}$, $a_{ii} = 0$. If $a_{ij} = 1$, we say j is the neighbor of i , and the set of all such j is the neighbor set of i , denoted by $\mathcal{N}(i)$. With the degree matrix

$$\mathcal{D} = \text{diag}(\text{deg}_1, \text{deg}_2, \dots, \text{deg}_N) \quad \text{where} \quad \text{deg}_i = \sum_{j=1}^N a_{ij},$$

Laplacian matrix of \mathcal{G} is defined by $L = \mathcal{D} - \mathcal{A}$. If there is a connection path between any pair of nodes, then \mathcal{G} is called connected. If the eigenvalues of L are denoted by $\lambda_1 \leq \lambda_2 \leq \dots \leq \lambda_N$, then \mathcal{G} is connected if and only if $\lambda_2 > 0$ [15]. In this paper, we use the undirected connected graph to describe the communication topology between multiple players.

A function $f: \mathbb{R}^n \rightarrow \mathbb{R}$ is said to be convex on \mathbb{R}^n when $f(\alpha x + (1-\alpha)x') \leq \alpha f(x) + (1-\alpha)f(x')$ is satisfied for all $x, x' \in \mathbb{R}^n$ and all $\alpha \in [0, 1]$.

A function $f: \mathbb{R}^n \rightarrow \mathbb{R}^n$ is L_f -Lipschitz ($L_f > 0$) on \mathbb{R}^n if $\|f(x) - f(x')\| \leq L_f \|x - x'\|$ is satisfied for all $x, x' \in \mathbb{R}^n$.

If there exists a $w > 0$ such that $(x - x')^\top (f(x) - f(x')) \geq w \|x - x'\|^2$ holds for all $x, x' \in \mathbb{R}^n$ and $x \neq x'$, the function $f: \mathbb{R}^n \rightarrow \mathbb{R}^n$ is w -strongly monotone. More details about the above definitions can be found in Ref. [16].

B. PH Model of PMSM

This paper considers N PMSMs communicating via a wireless network, which is described by an undirected graph \mathcal{G} . The node set is denoted by $\mathcal{V} = \{1, 2, \dots, N\}$. Then, by basic principles of electromagnetics such as voltage equation, the k -th PMSM ($k \in \mathcal{V}$) is modeled with the standard d - q model given as [13]

$$\begin{cases} L_{dk} \frac{di_{dk}}{dt} = -R_{sk} i_{dk} + \omega_k L_{qk} i_{qk} + V_{dk}, \\ L_{qk} \frac{di_{qk}}{dt} = -R_{sk} i_{qk} - \omega_k L_{dk} i_{dk} - \omega_k \phi_k + V_{qk}, \\ \tilde{J}_k \frac{d\omega_k}{dt} = n_k ((L_{dk} - L_{qk}) i_{dk} i_{qk} + \phi_k i_{qk}) - \tau_{lk} \end{cases} \quad (1)$$

where n_k is the number of pole pairs. L_{dk} and L_{qk} are stator inductances in d - q frame. R_{sk} is the stator winding resistance. τ_{lk} is a known constant load torque. ϕ_k is the d - q back electromotive force constant. \tilde{J}_k is the moment of inertia. ω_k is the angular velocity. i_{dk} and i_{qk} are currents in d - q axis. V_{dk} and V_{qk} are voltages in d - q axis.

The viscous friction is neglected in this model, because it is usually small. By defining the state vector as $x_k = \text{col}(x_{1k}, x_{2k}, x_{3k})$, where $x_{1k} = L_{dk} i_{dk}$, $x_{2k} = L_{qk} i_{qk}$, and $x_{3k} = (\tilde{J}_k / n_k) \omega_k$, the Hamiltonian (total energy) of Eq. (1) can be given as

$$H(x_k) = \frac{1}{2} \left(\frac{1}{L_{dk}} x_{1k}^2 + \frac{1}{L_{qk}} x_{2k}^2 + \frac{n_k}{\tilde{J}_k} x_{3k}^2 \right) \quad (2)$$

Based on the Hamiltonian function in Eq. (2), the system in Eq. (1) can be rewritten in PH form as

$$\dot{x}_k = (J_k(x_k) - R_k) \frac{\partial H(x_k)}{\partial x_k} + g_k u_k + \xi_k \quad (3)$$

with

$$g_k = \begin{bmatrix} 1 & 0 \\ 0 & 1 \\ 0 & 0 \end{bmatrix}, \quad u_k = \begin{bmatrix} V_{dk} \\ V_{qk} \end{bmatrix}, \quad \xi_k = \begin{bmatrix} 0 \\ 0 \\ -\frac{\tau_{lk}}{n_k} \end{bmatrix},$$

the structure matrix $J_k(x_k)$ and the dissipation matrix R_k being

$$J_k(x_k) = \begin{bmatrix} 0 & 0 & x_{2k} \\ 0 & 0 & -(x_{1k} + \phi_k) \\ -x_{2k} & x_{1k} + \phi_k & 0 \end{bmatrix},$$

$$R_k = \begin{bmatrix} R_{sk} & 0 & 0 \\ 0 & R_{sk} & 0 \\ 0 & 0 & 0 \end{bmatrix},$$

respectively.

III. PROBLEM FORMULATION

In this paper, we consider the speed synchronization problem of multiple PMSMs with the dynamic described by Eq. (3), as shown in Problem 1.

Problem 1 Design a controller u_k for the k -th PMSM ($k \in \mathcal{V}$) such that the rotor speed ω_k of the k -th PMSM asymptotically converges to the minimum point ω_k^* of the value function

$$V_k(\omega) = \frac{\alpha N}{2(N-1)}(\omega_k - \sigma(\omega(t)))^2 + \frac{1}{2}\beta(\omega_k - \omega_k(0))^2 \quad (4)$$

where $\omega = \text{col}(\omega_1, \omega_2, \dots, \omega_N)$, the average rotor speed of all PMSMs is denoted by

$$\sigma(\omega(t)) = \frac{1}{N} \sum_{k=1}^N \omega_k(t),$$

the initial speed of the k -th PMSM is denoted by $\omega_k(0)$, and the constant parameters $\alpha \gg \beta > 0$.

The value function in Eq. (4) is consistent with two parts.

(1) Each PMSM aims to achieve the speed synchronization with the control objective

$$\lim_{t \rightarrow \infty} \omega_k(t) - \sigma(\omega(t)) = 0, \quad k \in \mathcal{V} \quad (5)$$

(2) Considering from the perspectives of energy loss and system stability, each PMSM does not want the rotational speed to change too much from the initial value.

$$\lim_{t \rightarrow \infty} \omega_k(t) = \omega_k(0), \quad k \in \mathcal{V} \quad (6)$$

Since the rotor speed synchronization objective has more weight in this control problem, we define $\alpha \gg \beta$ to enlarge the weight of the first term in Eq. (4). Therefore, the rotor speed synchronous control problem is converted to Problem 1.

It is worth to notice that, in this optimization problem, only the rotor speed of the k -th PMSM ($k \in \mathcal{V}$) and the average rotor speed $\sigma(\omega(t))$ are required for the k -th PMSM. Since the rotor speed of other PMSMs $j \in \mathcal{V}/\{k\}$ is not available for the k -th PMSM because of sensitive state concerns, we define $\eta_k(t)$ as the estimation of $\sigma(\omega(t))$ for the k -th PMSM to design a controller that solves Problem 1.

It is easy to find that the value function $V_k(\omega)$ is continuously differentiable in ω and convex in ω_k when ω_j , $j \in \mathcal{V}/\{k\}$ is fixed.

Lemma 1 [17] The minimum point of Eq. (4) is ω^* if and only if

$$\nabla_{\omega_k} V_i(\omega^*) = 0, \quad k \in \mathcal{V} \quad (7)$$

Hence, our task is to design a distributed control algorithm for the k -th PMSM ($k \in \mathcal{V}$) to solve Problem 1.

IV. MAIN RESULT

Before giving the distributed controller, the following mappings are defined as

$$F_k(\omega_k, \sigma(\omega)) := \nabla_{\omega_k} V_k(\omega), \quad (8)$$

$$G_k(\omega_k, \eta_k) := F_k(\omega_k, \sigma(\omega))|_{\sigma(\omega)=\eta_k}$$

where η_k is the estimation of the k -th PMSM to the average rotor speed $\sigma(t)$.

Furthermore, we can define the following vectors

$$F(\omega) := \text{col}(\nabla_{\omega_1} V_1(\omega), \nabla_{\omega_2} V_2(\omega), \dots, \nabla_{\omega_N} V_N(\omega)), \quad (9)$$

$$G(\omega, \eta) := \text{col}(G_1(\omega_1, \eta_1), G_2(\omega_2, \eta_2), \dots, G_N(\omega_N, \eta_N))$$

where $\eta = \text{col}(\eta_1, \eta_2, \dots, \eta_N) \in \mathbb{R}^N$.

Based on these mappings, the distributed controller of the k -th PMSM is defined as

$$\begin{cases} u_k = -A_k \begin{bmatrix} \frac{\partial H(x_k)}{\partial x_{1k}} \\ \frac{\partial H(x_k)}{\partial x_{2k}} \end{bmatrix} - B_k \omega_k + C_k, \\ \dot{\eta}_k = s_k, \\ \dot{s}_k = -k_2 s_k - \rho(\eta_k - \omega_k) - \sum_{j=1}^N a_{kj}((\eta_k - \eta_j) + (v_k - v_j)), \\ \dot{v}_k = \sum_{j=1}^N a_{kj}(\eta_k - \eta_j) + \sum_{j=1}^N a_{kj}(s_k - s_j) \end{cases} \quad (10)$$

where the matrix $A_k = \text{diag}(-R_{sk}, -R_{sk})$, the matrix $B_k = \text{col}(x_{2k}, -(x_{1k} + \phi_k))$, and the matrix $C_k = \text{col}(0, k_1(-k_2 \dot{\omega}_k - L_{qk} \dot{J}_k G_k(\omega_k, \eta_k)))$. Constants $k_1 = \frac{L_{qk} \dot{J}_k}{((L_{dk} - L_{qk})i_{dk}(0) + \phi_k)n_k}$ and $k_2 > 0$ are given in Theorem 1. We assume that the initial value of each agent satisfies $i_{dk}(0) \neq \frac{\phi_k}{L_{qk} - L_{dk}}$, $k \in \mathcal{V}$.

Remark 1 The condition $i_{dk}(0) \neq \frac{\phi_k}{L_{qk} - L_{dk}}$ is a value limitation on the initial value of i_{dk} with the constant parameters ϕ_k , L_{qk} , and L_{dk} of the i -th PMSM ($i \in \mathcal{V}$). In the surface-mounted PMSM, $L_{qk} - L_{dk} = 0$, this condition will always satisfy. If the initial value $i_{dk}(0) = \frac{\phi_k}{L_{qk} - L_{dk}}$, then we can apply a small perturbation to make i_{dk} change slightly and use the controller in Eq. (10).

In the distributed controller in Eq. (10), the first equation is designed to relate the coupled-state relationship of the PH system in Eq. (3), the estimation η_k of the k -th PMSM ($k \in \mathcal{V}$) is updated by the estimation from its neighbours $j \in \mathcal{N}(k)$, and s_k and v_k are auxiliary variables to ensure the accuracy of the estimation.

By inserting x_k , A_k , B_k , and C_k , we have

$$u_k = \begin{bmatrix} R_{sk} i_{dk} - L_{qk} i_{qk} \omega_k \\ R_{sk} i_{qk} + (L_{dk} i_{dk} + \phi_k) \omega_k + k_1(-k_2 \dot{\omega}_k - G_k(\omega_k, \eta_k)) \end{bmatrix} \quad (11)$$

Theorem 1 shows the convergence of the closed-loop system (the PH system in Eq. (3) under the action of the distributed controller in Eq. (10)).

Theorem 1 The PH system in Eq. (3) of the k -th PMSM ($k \in \mathcal{V}$) is exponentially convergent to the equilibrium under the action of the distributed controller in Eq. (10) with $k_2 > 1 + \lambda_N/4 + L_f^2/\gamma$, where $L_f > 0$ and $\gamma > 0$ are the Lipschitz constant and the strongly monotone parameter of mapping $\Xi: \mathbb{R} \times \mathbb{R} \rightarrow \mathbb{R}^2$, respectively.

Proof Consider the multi-agent system in Eq. (3), taking the derivative of x_{3k} with respect to time, we have

$$\dot{\omega}_k = \frac{n_k}{J_k} (-\dot{x}_{2k} i_{dk} - x_{2k} \dot{i}_{dk} + \dot{x}_{1k} i_{qk} + (x_{1k} + \phi_k) \dot{i}_{qk}) \quad (12)$$

By inserting the distributed controller in Eq. (10) into the PH model in Eq. (3) of the k -th PMSM ($k \in \mathcal{V}$), we obtain

$$\begin{cases} \dot{i}_{dk} = 0, \\ \dot{i}_{qk} = \frac{k_1}{L_{qk}} (-k_2 \dot{\omega}_k - G_k(\omega_k, \eta_k)), \\ \dot{\eta}_k = s_k, \\ \dot{s}_k = -k_2 s_k - \rho(\eta_k - \omega_k) - \sum_{j=1}^N a_{kj}((\eta_k - \eta_j) + (v_k - v_j)), \\ \dot{v}_k = \sum_{j=1}^N a_{kj}(\eta_k - \eta_j) + \sum_{j=1}^N a_{kj}(s_k - s_j) \end{cases} \quad (13)$$

Combining Eqs. (12) and (13), we have

$$\begin{cases} \dot{\omega}_k = -k_2 \dot{\omega}_k - G_k(\omega_k, \eta_k), \\ \dot{\eta}_k = s_k, \\ \dot{s}_k = -k_2 s_k - \rho(\eta_k - \omega_k) - \sum_{j=1}^N a_{kj}((\eta_k - \eta_j) + (v_k - v_j)), \\ \dot{v}_k = \sum_{j=1}^N a_{kj}(\eta_k - \eta_j) + \sum_{j=1}^N a_{kj}(s_k - s_j), k \in \mathcal{V} \end{cases} \quad (14)$$

Define an L_f -Lipschitz and γ -strongly monotone map as

$$\Xi = \begin{bmatrix} G(\omega, \eta) \\ \rho(\eta - \omega) \end{bmatrix} \quad (15)$$

where $\eta = \text{col}(\eta_1, \eta_2, \dots, \eta_N)$.

Let $\tilde{\omega} = \text{col}(\omega, \eta)$, $\tilde{s} = \text{col}(\dot{\omega}, s)$, $v = \text{col}(v_1, v_2, \dots, v_N)$, $\dot{\omega} = \text{col}(\dot{\omega}_1, \dot{\omega}_2, \dots, \dot{\omega}_N)$, and $s = \text{col}(s_1, s_2, \dots, s_N)$. By virtue of the definition of Eq. (15) and writing the closed-loop system in Eq. (14) in a compact form, we have

$$\begin{cases} \dot{\tilde{\omega}} = \tilde{s}, \\ \dot{\tilde{s}} = -k_2 \tilde{s} - \Xi(\tilde{\omega}) - \Phi_1 \tilde{\omega} - \Phi_2 v, \\ \dot{v} = \Phi_3(\tilde{\omega} + \tilde{s}) \end{cases} \quad (16)$$

where

$$\Phi_1 = \begin{bmatrix} 0 & 0 \\ 0 & L \end{bmatrix}, \Phi_2 = \begin{bmatrix} 0 \\ L \end{bmatrix}, \Phi_3 = \begin{bmatrix} 0 & L \end{bmatrix}.$$

With the following coordinate transformation, we have

$$\begin{cases} \tilde{\omega} = \text{col}(\tilde{\omega}_q, \tilde{\omega}_y) = \tilde{\omega} - \tilde{\omega}^*, \\ \tilde{s} = \text{col}(\tilde{s}_q, \tilde{s}_y) = \tilde{s} - \tilde{s}^*, \\ \tilde{v} = v - v^* \end{cases} \quad (17)$$

where $\tilde{\omega}_q, \tilde{\omega}_y, \tilde{s}_q$, and $\tilde{s}_y \in \mathbb{R}^N$. Then the following system is obtained by Eq. (16)

$$\begin{cases} \dot{\tilde{\omega}} = \tilde{s}, \\ \dot{\tilde{s}} = -k_2 \tilde{s} - h - \Phi_1 \tilde{\omega} - \Phi_2 \tilde{s}, \\ \dot{\tilde{v}} = \Phi_3(\tilde{\omega} + \tilde{s}) \end{cases} \quad (18)$$

where $h = \Xi(\tilde{\omega}) - \Xi(\tilde{\omega}^*)$.

Since $\tilde{\omega} \rightarrow \tilde{\omega}^*$ is equivalent to $\tilde{\omega} \rightarrow 0$, we are going to analyze the convergence of $\tilde{\omega} \rightarrow 0$.

With the eigenvalue decomposition of Laplace matrices, we have

$$\begin{cases} \text{col}(\omega_{qr}, \omega_{qR}) = [r \quad R]^T \tilde{\omega}_q, \\ \text{col}(\omega_{yr}, \omega_{yR}) = [r \quad R]^T \tilde{\omega}_y, \\ \text{col}(s_{qr}, s_{yR}) = [r \quad R]^T \tilde{s}_q, \\ \text{col}(s_{yr}, s_{sR}) = [r \quad R]^T \tilde{s}_y, \\ \text{col}(\delta_1, \delta_2) = [r \quad R]^T \tilde{v} \end{cases} \quad (19)$$

where $\omega_{qr}, \omega_{yr}, s_{qr}, s_{yr}$, and $\delta_1 \in \mathbb{R}$, $\omega_{qR}, \omega_{yR}, s_{qR}, s_{yR}$, and $\delta_2 \in \mathbb{R}^{N-1}$, $r = \frac{1}{\sqrt{N}} \mathbf{1}_N$, $RR^T = I_N - \frac{1}{N} \mathbf{1}_N \mathbf{1}_N^T$, $r^T R = \mathbf{0}_{N-1}^T$, and $R^T R = I_{N-1}$.

Let $\omega_r = \text{col}(\omega_{qr}, \omega_{yr})$, $\omega_R = \text{col}(\omega_{qR}, \omega_{yR})$, $s_r = \text{col}(s_{qr}, s_{yr})$, and $s_R = \text{col}(s_{qR}, s_{yR})$, and then we rewrite Eq. (18) as

$$\begin{cases} \dot{\omega}_r = s_r, \\ \dot{s}_r = -k_2 s_r - \begin{bmatrix} r & 0 \\ 0 & r \end{bmatrix}^T h, \\ \dot{\delta}_1 = 0_{N-1} \end{cases} \quad (20)$$

$$\begin{cases} \dot{\omega}_R = s_R, \\ \dot{s}_R = -k_2 s_R - \begin{bmatrix} R & 0 \\ 0 & R \end{bmatrix}^T h - \begin{bmatrix} 0 \\ R^T L R \end{bmatrix} (\omega_R - \delta_2), \\ \dot{\delta}_2 = \begin{bmatrix} 0 & R^T L R \end{bmatrix} \omega_R + \begin{bmatrix} 0 & R^T L R \end{bmatrix} s_R \end{cases} \quad (21)$$

Take the following candidate Lyapunov function as

$$L = \frac{1}{2} (\|\omega_r + s_r\|^2 + (k_2 - 1) \|\omega_r\|^2 + \|\omega_R + s_R\|^2 + (k_2 - 1) \|\omega_R\|^2 + \|\delta_2\|^2 + \theta \|s_{yR} + \delta_2\|^2) \quad (22)$$

where $0 < \theta < \min \left\{ \frac{2\lambda_2 k_2 \gamma}{2k_2 + \lambda_2}, \frac{2\lambda_2 \left(k_2 - \frac{L_f^2}{\gamma} - \frac{1}{4} \lambda_N - 1 \right)}{k_2(k_2 + 1) + 2\lambda_2 \lambda_N} \right\}$ and

$k_2 > L_f^2/\gamma + \lambda_N/4 + 1$.

Let $\tilde{r} = \begin{bmatrix} r & 0 \\ 0 & r \end{bmatrix}$ and $\tilde{R} = \begin{bmatrix} R & 0 \\ 0 & R \end{bmatrix}$, then the derivative of

L_y along Eqs. (20) and (21) is

$$\begin{aligned} \dot{L}_y = & -\omega_r^T \tilde{r}^T h - \omega_R^T \tilde{R}^T h - (k_2 - 1) \|s_r\|^2 - (k_2 - 1) \|s_R\|^2 - \\ & s_r^T \tilde{r}^T h - s_R^T \tilde{R}^T h - \omega_{yR}^T R^T L R \omega_{yR} - s_{yR}^T R^T L R \omega_{yR} + \\ & \theta (-k_2 \delta_2^T s_{yR} - \delta_2^T [0 \quad R^T] h - \delta_2^T R^T L R \delta_2 + \\ & s_{yR}^T R^T L R s_{yR} - k_2 \|s_{yR}\|^2 - s_{yR}^T [0 \quad R^T] h) \end{aligned} \quad (23)$$

With the definition of $\Xi(\omega, \eta)$ in Eq. (15) and F_k and G_k in Eq. (8), it is easy to find that the mapping $\Xi(\omega, \eta)$ is γ -strongly monotone and L_f -Lipschitz for some $\rho > 0$ on \mathbb{R}^2 , and $G(\omega, \eta)$ is L_g -Lipschitz. Hence, we have

$$\omega_r^T \tilde{r}^T h + \omega_R^T \tilde{R}^T h \geq \gamma(\|\omega_r\|^2 + \|\omega_R\|^2) \quad (24)$$

and

$$s_r^T \tilde{r}^T h + s_R^T \tilde{R}^T h \leq \frac{1}{2} \left(\frac{L_f^2}{\gamma} (\|s_r\|^2 + \|s_R\|^2) + \gamma(\|\omega_r\|^2 + \|\omega_R\|^2) \right) \quad (25)$$

With the Schur complement lemma, the matrix

$$\begin{bmatrix} R^T L R & \frac{1}{2} R^T L R \\ \frac{1}{2} R^T L R & \frac{1}{4} \lambda_N I \end{bmatrix} \geq 0 \quad (26)$$

implies that

$$\begin{aligned} & \omega_{yR}^T R^T L R \omega_{yR} + s_{yR}^T R^T L R \omega_{yR} = \\ & \begin{bmatrix} \omega_{yR}^T & s_{yR}^T \end{bmatrix} \begin{bmatrix} R^T L R & \frac{1}{2} R^T L R \\ \frac{1}{2} R^T L R & \frac{1}{4} \lambda_N I \end{bmatrix} \begin{bmatrix} \omega_{yR} \\ s_{yR} \end{bmatrix} - \frac{1}{4} \lambda_N \|s_{yR}\|^2 \geq \\ & - \frac{1}{4} \lambda_N \|s_R\|^2 \end{aligned} \quad (27)$$

In addition, with $ab \leq \frac{c}{2} a^2 + \frac{1}{2c} b^2$, $c > 0$, we have

$$-k_2 \delta_2^T s_{yR} \leq \frac{1}{2} k_2 \left(\frac{\lambda_2}{k_2 + 1} \|\delta_2\|^2 + \frac{k_2 + 1}{\lambda_2} \|s_{yR}\|^2 \right) \quad (28)$$

$$-\delta_2^T [0 \quad R^T] h \leq \frac{1}{2} \left(\lambda_2 \|\delta_2\|^2 + \frac{1}{\lambda_2} (\|\omega_r\|^2 + \|\omega_R\|^2) \right) \quad (29)$$

$$-s_{yR}^T [0 \quad R^T] h \leq \frac{1}{2} \left(2k_2 \|s_{yR}\|^2 + \frac{1}{2k_2} (\|\omega_r\|^2 + \|\omega_R\|^2) \right) \quad (30)$$

With Formulas (24)–(30), the derivative of L_y satisfies

$$\begin{aligned} \dot{L}_y \leq & - \left(\frac{1}{2} \gamma - \theta \left(\frac{1}{2\lambda_2} + \frac{1}{4k_2} \right) \right) (\|\omega_r\|^2 + \|\omega_R\|^2) - \\ & \left(k_2 - \frac{L_f^2}{\gamma} - 1 \right) \|s_r\|^2 - \left(k_2 - \frac{L_f^2}{\gamma} - 1 - \frac{\lambda_N}{4} - \right. \\ & \left. \theta \left(\lambda_N + \frac{k_2(k_2 + 1)}{2\lambda_2} \right) \right) \|s_R\|^2 - \frac{\theta \lambda_2}{2(k_2 + 1)} \|\delta_2\|^2 \end{aligned} \quad (31)$$

Since L_y and its derivative \dot{L}_y are quadratic, and all states occur in L_y and \dot{L}_y , the rotor speed ω exponentially converges to ω^* . ■

Furthermore, we aim to prove that the equilibrium ω^* , to which the closed-loop system in Eq. (16) converges, is the minimum point of the value function $V_k(\omega)$ in Problem 1.

Theorem 2 If ω^* is a minimum point of the value function $V_k(\omega)$ defined in Problem 1, then there are η^* , s^* , and $v^* \in \mathbb{R}^N$ such that $(\omega^*, \eta^*, s^*, v^*)$ is an equilibrium of multi-agent systems with PH dynamics in Eq. (3) under the action of the distributed controller in Eq. (10). Conversely, if $(\omega^*, \eta^*, s^*, v^*)$ is an equilibrium of multi-agent systems with PH dynamics in Eq. (3) under the control of the distributed controller in Eq. (10), then ω^* is a minimum point of the value function $V_k(\omega)$.

Proof Taking Eq. (16) at its equilibrium, we have

$$\begin{cases} \tilde{s}^* = 0_{2N}, \\ -k\tilde{s}^* - \Xi(\tilde{\omega}^*) - \Phi_1 \tilde{\omega}^* - \Phi_2 v^* = 0_{2N}, \\ \Phi_3(\tilde{\omega}^* + \tilde{s}^*) = 0_{2N} \end{cases} \quad (32)$$

which indicates that

$$\begin{cases} L\eta^* = 0_N, \\ G(\omega^*, \eta^*) = 0_N, \\ \rho(\eta^* - \omega^*) + Lv^* = 0_N \end{cases} \quad (33)$$

Given that the undirected connected graph \mathcal{G} has the property that its Laplace matrix L satisfies $1_N^T L = 0_N^T$, we multiply both sides of the third equation of Eq. (33) by 1_N^T from the left, then we obtain

$$\eta_k^* = \frac{1}{N} \sum_{j=1}^N \omega_j^* \quad (34)$$

and meanwhile we have

$$G_k(\omega_k^*, \eta_k^*) = F_k(\omega_k^*, \sigma(\omega^*)) = 0 \quad (35)$$

for any $k \in \mathcal{V}$.

Combing with the definition of $F_k(\omega_k, \sigma(\omega))$ in Eq. (8), with Formulas (30) and (34), one can get

$$\nabla_{\omega_k} V_k(\omega^*) = 0, \quad \forall k \in \mathcal{V} \quad (36)$$

and this indicates that ω^* is the minimum point of the value function defined in Problem 1.

On the other hand, if ω^* is a minimum point of the value function defined in Problem 1, then $F_k(\omega_k^*, \sigma(\omega^*)) = 0_m$. By choosing

$$\eta_k^* = \sigma(\omega^*), \quad k \in \mathcal{V} \quad (37)$$

we have

$$G_k(\omega_k^*, \eta_k^*) = 0 \quad (38)$$

In addition, taking any $\hat{r} \in \mathbb{R}$ and letting $r' = 1_N \hat{r}$, we have

$$(\eta^* - \omega^*)^T r' = 0 \quad (39)$$

By $1_N^T L = 0_N^T$, we have $r' \in \ker(L)$. With the orthogonal decomposition of zero space and range space, it can be obtained that $\rho(\eta^* - \psi(\omega^*)) \in \text{range}(L)$, where $\rho > 0$. Therefore, there is v^* such that

$$\rho(\eta^* - \psi(\omega^*)) + Lv^* = 0_N \quad (40)$$

Finally, when we choose $\tilde{s}^* = 0_{2N}$, the proof is completed. ■

By using Theorems 1 and 2, it can be concluded that the PH system in Eq. (3) exponentially converges to the equilibrium ω^* with the distributed controller in Eq. (10). This equilibrium corresponds to the minimum point of the value function defined in Problem 1. At this point, the rotor speeds of N PMSMs achieve a consensus.

V. SIMULATION

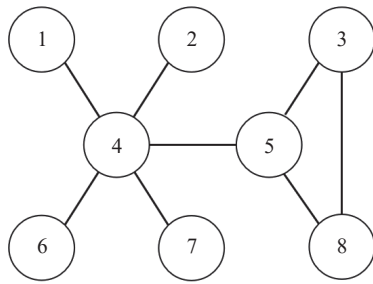
In this section, a simulation example is given to illustrate the effectiveness of the proposed distributed controller in Eq. (10).

Consider 8 PMSMs described by Eq. (3) communicating over a wireless network, which is characterized by an undirected connected graph \mathcal{G} as depicted in Fig. 1(a).

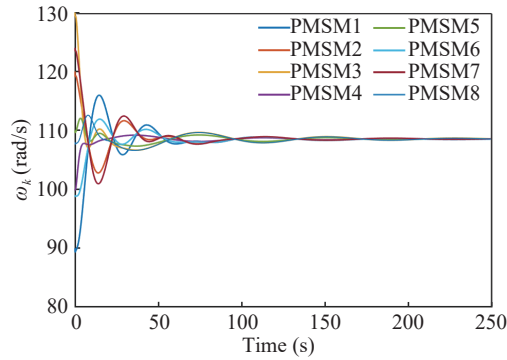
As described in Section III, each PMSM has two goals and the value function of the k -th PMSM ($k \in [1:8]$) is designed as

$$V_k(\omega) = \frac{40}{7}(\omega_k - \sigma(\omega))^2 + 0.05(\omega_k - \omega_k(0))^2 \quad (41)$$

Let $R_{sk} = 1 \Omega$, $\tau_{lk} = 2 \text{ Nm}$, $i_{dk}(0) = 0 \text{ A}$, and $k \in [1:8]$, and other parameters are assignment in Table 1. The initial conditions for 8 PMSMs are all different. Then the simulation results and the control input are shown in Fig. 1(b) and Fig. 2. As illustrated in Fig. 1(b), the rotor speeds of the 8 PMSMs eventually converge to unity, achieving synchronised speed control across all PMSMs. This result demonstrates the effectiveness of the proposed distributed controller.



(a) Communication topology



(b) Evolution of rotor speed

Figure 1 Communication topology and evolution of rotor speed of 8 PMSMs.

Table 1 Parameter table.

k	n_k	ϕ_k (Vs)	L_{qk} (mH)	L_{dk} (mH)	\tilde{J}_k (kg·m ²)	$\omega_k(0)$ (rad/s)	$i_{qk}(0)$ (A)
1	2	0.25	5.45×10^{-3}	4.48×10^{-3}	0.20	90	1.1
2	4	0.14	5.40×10^{-3}	5.24×10^{-3}	0.15	120	1.2
3	2	0.15	5.50×10^{-3}	5.15×10^{-3}	0.24	130	1.3
4	4	0.20	5.80×10^{-3}	5.18×10^{-3}	0.18	100	2.1
5	2	0.24	5.65×10^{-3}	4.65×10^{-3}	0.22	110	2.2
6	4	0.18	5.35×10^{-3}	4.45×10^{-3}	0.26	99	1.8
7	2	0.20	5.50×10^{-3}	4.44×10^{-3}	0.24	124	1.6
8	4	0.22	5.70×10^{-3}	5.20×10^{-3}	0.17	108	1.5

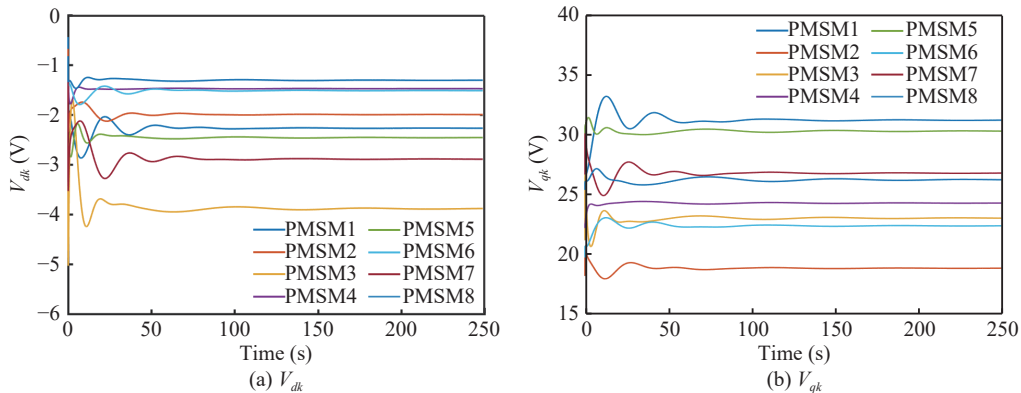


Figure 2 Control input of 8 PMSMs.

corresponds to the minimum of the value function. Finally, a simulation example is provided to demonstrate the effectiveness of the proposed distributed controller.

B. Future Work

In this paper, only undirected connected graph is considered. Next, we will investigate some more broader topologies with time-delay or packet loss issues. Additionally, addressing the challenges posed by unknown torques and model uncertainties could be valuable in the future.

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