Adaptive Dynamic Event-Triggered Control for Strict-Feedback Nonlinear System with Time-Varying Parameters

Yu Liu, Yongchao Liu, and Haiyang Chen

***Abstract*—In this paper, an adaptive dynamic event-triggered asymptotic control scheme is designed for strict-feedback systems with time-varying parameters. The congelation of variables technique is employed to address the time-varying parameters in the system. During the controller design, two parameter estimation adaptive laws are constructed. The tuning function is introduced in the design process to avoid over-parameterization. To further conserve communication resources, a dynamic event-triggered approach is proposed, in which a non-negative variable is introduced to update the threshold parameter dynamically. The global uniform asymptotic stability of the closed-loop system is proven through the Lyapunov stability analysis. The feasibility of the scheme is demonstrated by simulation.**

***Index Terms*—Adaptive control, nonlinear system, dynamic event-triggered control, time-varying parameter**

I. Introduction

ADAPTIVE control is a widely used approach in control systems. In recent decades, the adaptive control method has been widely studied and applied [1–3]. It is widely known that nonlinearity is prevalent in most practical engineering systems, and its presence poses significant challenges for controller design and synthesis. Consequently, control issues related to nonlinear systems have garnered considerable attention. In Ref. [4], an adaptive nonlinear control technique is developed to reduce the dynamic order of the controller, thereby improving system stability. In Ref. [5], a robust recursive design method is introduced for uncertain nonlinear system (UNS) with a vector strict-feedback form. In Ref. [6], Nussbaum function and barrier Lyapunov function are introduced to address uncertain control gains in UNS. To overcome matching conditions in nonlinear systems, the adaptive backstepping technique has been introduced and extensively studied [7–10]. In Ref. [7], an improved adaptive backstepping design approach is developed for UNS. In Ref. [8], two new control schemes based on the backstepping method are proposed for UNS. In Ref. [9], a controller is developed for UNS by using an optimization backstepping technique. In Ref. [10], an adaptive neural strategy is presented for UNS to overcome the challenge of unknown control direction. However, it is important to note that the above studies mainly focus on systems with constant parameters.

In practical applications, system parameters change with time [11, 12]. The time-varying parameters directly influence some dynamic properties of the system. Consequently, the control problems of time-varying system have received increasing attention over the last decade. In Ref. [13], a parallel control approach is suggested for zero-sum games in the context of unknown time-varying systems. In Ref. [14], the challenges of unknown time-varying parameters in the system are overcome by integrating bounded estimation techniques and smooth functions. In Ref. [15], a control strategy based on a special funnel function is introduced for time-varying nonlinear complex systems. In Ref. [16], the stability of nonlinear time-varying system (NTS) is proven using a Lyapunov function that includes a lower bound on the control gain. In Ref. [17], an enhanced Nussbaum function is applied to address the strict-feedback NTS. The aforementioned studies address time-varying parameters by estimating their upper bounds. This approach can result in overly conservative controller design or increased system complexity. To directly address time-varying parameters in the system, the congelation of variables method is proposed in Ref. [18]. This method replaces the unknown time-varying parameters in the Lyapunov design with the unknown constant parameters, thus relaxing the general time-varying parameter change rate limitation. This method can additionally be integrated with the passive-based approach [19] and the adaptive immersion and invariance scheme [20]. In Ref. [21], an adaptive backstepping control method is introduced for UNS with time-varying parameters by utilizing nonlinear mapping techniques and the congelation of variables approach. In Ref. [22], the congelation of variables approach is used to develop an effective control strategy for NTS.

The aforementioned control methods are based on periodic sampling. However, this control method demands periodic updates to the signal, which causes excessive unnecessary computational burden and higher transmission expenses. To overcome the problem, the event-triggered control (ETC)

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method is introduced. The fundamental idea of ETC is to transmit signals through the communication channel only when specific preset conditions are triggered. The event-triggered mechanism (ETM) is more flexible than the time-triggered mechanism, which reduces the burden of network communication while achieving the desired goal and stability standard. Therefore, ETC has received attention from the academic circles in improving resource utilization. In recent decades, there has been considerable research on ETM [23–29]. In Ref. [23], an ETM is added to the design process of an asymptotic tracking controller for nonlinear systems. In Ref. [24], an ETM based on relative thresholds is introduced to reduce data release. For UNS with time-varying parameters, a strategy combining an ETM and a prescribed-time control is proposed [25]. In Ref. [26], a novel control approach is presented to address the challenge of uncertain control directions in the system by incorporating ETM and adaptive technique. In Ref. [27], an ETM is added to the adaptive asymptotic control design of nonlinear system to reduce unnecessary control updates. In Ref. [28], an ETC is proposed for nonlinear multiagent systems to reduce the communication resource consumption. However, the above event-triggered schemes are static, that is to say, the threshold parameters of the ETM are fixed. To further accelerate the utilization of network resources, the dynamic event-triggered mechanism (DETM) is introduced. DETM can dynamically update the threshold parameter, and therefore it can further reduce unnecessary resource waste. In Ref. [30], an adaptive control approach is presented, which combines an observer with a DETM to manage UNS. In Ref. [31], a DETM is introduced for UNS, which saves communication resources. In Ref. [32], two independent ETMs are designed for a class of zero-sum game problems to reduce communication and computation frequency. In Ref. [33], a control strategy integrating adaptive critic and ETC is proposed for a class of non-affine systems, which effectively reduces the consumption of computational resources.

In summary, there has been substantial advancement in the development of adaptive ETC strategies for nonlinear systems. However, the adaptive control problem for NTS based on DETM has not been fully studied, and some important problems still need to be explored. Therefore, this paper proposes a dynamic event-triggered approach for time-varying systems to reduce data transmission. The primary contributions are summarized as follows:

(1) To address time-varying parameters of UNS, the congelation of variables approach is integrated into the backstepping framework. In addition, the tuning function is introduced in the adaptive control design process to avoid over-parameterization.

(2) A dynamic ETC scheme is devised for strict-feedback nonlinear systems with unknown boundary time-varying parameters. The design introduces a dynamic threshold parameter to realize the dynamic adjustment of the event-triggered threshold. The proposed scheme ensures the asymptotic stability of the closed-loop system while reducing the communication burden.

(3) Compared with uniform bounded control results [30], our established controller can achieve global uniform asymptotic stability. Although Ref. [34] achieves asymptotic stability, the parameters of the system are constants. The presented dynamic ETC method not only tackles the issue of time-varying parameters, but also ensures the asymptotic convergence of the system.

Notation For a vector v, $\|v\|$ represents the Euclidean 2-norm. For an $n\times m$ matrix W, $\|W\|_{\mathrm{F}}=\sqrt{\sum_{i=1}^{n}\sum_{j=1}^{m}W_{ij}}$ represents the Frobenius norm, where W_{ij} denotes the i-th element on the j-th column. For an n-dimensional time-varying signal $s:\mathbb{R}\to\mathbb{R}^n$, with its image contained within a compact set S, $\delta_s\in\mathbb{R}$ represents the maximum value of the 2-norm of s, i.e., $\delta_s=\sup_{t\geqslant 0}|s(t)|\geqslant 0$. $\Gamma>0$ is a symmetric matrix. $\mathrm{sgn}(\cdot)$ is the sign function.

II. Problem Statement

Consider the strict-feedback nonlinear system

$$\begin{cases}\dot{x}_1=\phi_1^{\mathrm{T}}(x_1)\theta(t)+x_2,\\ \dot{x}_2=\phi_2^{\mathrm{T}}(\underline{x}_2)\theta(t)+x_3,\\ \vdots\\ \dot{x}_i=\phi_i^{\mathrm{T}}(\underline{x}_i)\theta(t)+x_{i+1},\\ \vdots\\ \dot{x}_n=\phi_n^{\mathrm{T}}(\underline{x}_n)\theta(t)+b(t)u(t)\end{cases}\tag{1}$$

where $\underline{x}_i(t)=[x_1,x_2,\dots,x_i]^{\mathrm{T}}\in\mathbb{R}^i$, $i=2,3,\dots,n$ is system state, $u(t)\in\mathbb{R}$ is the input, $\theta(t)\in\mathbb{R}^q$ is the vector of unknown time-varying parameters, and $b(t)\in\mathbb{R}$ is an unknown time-varying parameter. The regressor $\phi_i:\mathbb{R}^i\to\mathbb{R}^q$ $(i=1,2,\dots,n)$ is smooth mapping and satisfies $\phi_i(0)=0$.

The objective of this paper is to develop a control strategy for Eq. (1), which can effectively handle the impact of time-varying parameters and ensure the stability of the system.

To achieve this objective, we make Assumptions 1 and 2.

Assumption 1 The parameter θ is piecewise continuous and $\theta(t)\in\Theta_0$, for all $t>0$, where Θ_0 is a compact set. Suppose that the "radius" of Θ_0, represented by δ_{Δ_θ}, is known.

Assumption 2 [27] The control direction is fixed and known. There are constants l_b and $\bar{b}$, where l_b is unknown and $\bar{b}$ is known satisfying $0<|l_b|\leqslant|b(t)|\leqslant\bar{b}$.

Remark 1 For any practical system, Assumption 1 can be easily satisfied in real-world applications [18]. The known "radius" δ_{Δ_θ} implies that the variation range of time-varying parameters is known. Specifically, from Assumption 1, for any $\theta(t)\geqslant 0$, there exist two unknown constant vectors $\theta_1(t)$ and $\theta_2(t)$ such that $\theta_1(t)\leqslant\theta(t)\leqslant\theta_2(t)$, then the "radius" of Θ_0 is $\delta_{\Delta_\theta}=\|\theta_1(t)-\theta_2(t)\|/2$, which is assumed to be known. A larger δ_{Δ_θ} value indicates greater allowable uncertainty in Eq. (1). Assumption 2 is slightly more restrictive than that in Ref. [18], as it requires the time-varying parameter $b(t)$ to have a known upper bound, which will be used to handle the sampling errors caused by the event-triggered mechanism.

Lemma 1 Given any smooth function $\sigma(t):[0,\infty)\to(0,\infty)$, Formula (2) is established

$$|s|-\frac{s^2}{\sqrt{s^2+\sigma^2}}\leqslant\sigma,\ s\in\mathbb{R} \tag{2}$$

III. DESIGN OF EVENT-TRIGGERED CONTROLLER

Define error variables

$$\begin{aligned}&z_1=x_1,\\&z_i=x_i-\alpha_{i-1},\ i=2,3,\ldots,n\end{aligned} \tag{3}$$

where z_i is the error variable, and α_{i-1} is the virtual control law.

Step 1: Differentiating z_1 yields

$$\dot z_1=\dot x_1=x_2+\phi_1^{\mathrm T}\theta=z_2+\alpha_1+\phi_1^{\mathrm T}\hat\theta+\phi_1^{\mathrm T}(l_\theta-\hat\theta)+\phi_1^{\mathrm T}\varDelta_\theta \tag{4}$$

where $\varDelta_\theta=\theta-l_\theta$.

Define the Lyapunov function V_1 as

$$V_1=\frac12 z_1^2+\frac12(l_\theta-\hat\theta)^{\mathrm T}\varGamma^{-1}(l_\theta-\hat\theta) \tag{5}$$

where l_θ can be considered as the average of $\theta(t)$.

Taking the derivative of V_1, we get

$$\begin{aligned}\dot V_1=\;&z_1(z_2+\alpha_1+\phi_1^{\mathrm T}\hat\theta+\phi_1^{\mathrm T}(l_\theta-\hat\theta)+\phi_1^{\mathrm T}\varDelta_\theta)-\\&(l_\theta-\hat\theta)^{\mathrm T}\varGamma^{-1}\dot{\hat\theta}=\\&z_1z_2+z_1(\alpha_1+\omega_1^{\mathrm T}\hat\theta)+z_1\omega_1^{\mathrm T}\varDelta_\theta-\\&(l_\theta-\hat\theta)^{\mathrm T}(\varGamma^{-1}\dot{\hat\theta}-\omega_1z_1)\end{aligned} \tag{6}$$

where $\omega_1=\phi_1$.

By utilizing the Hadamard's lemma [35], we can deduce that

$$\omega_i=\bar W_i\underline{z}_i \tag{7}$$

where $\underline{z}_i=[z_1,z_2,\ldots,z_i]^{\mathrm T}$, and $W_i\in\mathbb{R}$ represents the smooth mapping.

Therefore, applying the Young's inequality, we can derive

$$z_1\omega_1^{\mathrm T}\varDelta_\theta=z_1\varDelta_\theta^{\mathrm T}\bar W_1z_1\leqslant\frac12\left(\frac{\delta_{\varDelta_\theta}}{\varepsilon_{\varDelta_\theta}}+\delta_{\varDelta_\theta}\varepsilon_{\varDelta_\theta}\|\bar W_1\|_{\mathrm F}^2\right)z_1^2 \tag{8}$$

where $\varepsilon_{\varDelta_\theta}>0$ is a constant, $\delta_{\varDelta_\theta}=\sup\limits_{t\geqslant0}|\theta(t)|\geqslant0$.

Next, we design the virtual control law α_1 and the tuning function τ_1 as

$$\begin{aligned}\alpha_1=&-c_1z_1-\frac n2\frac{\delta_{\varDelta_\theta}}{\varepsilon_{\varDelta_\theta}}z_1-\frac12\delta_{\varDelta_\theta}\varepsilon_{\varDelta_\theta}\|\bar W_1\|_{\mathrm F}^2z_1-\\&\omega_1^{\mathrm T}\hat\theta-\frac12\frac1{\varepsilon_{\bar\psi}}z_1\end{aligned} \tag{9}$$

$$\tau_1=\omega_1z_1 \tag{10}$$

where $c_1>0$ and $\varepsilon_{\bar\psi}>0$.

Substituting Eqs. (9) and (10) into Eq. (6), we get

$$\begin{aligned}\dot V_1\leqslant&-c_1z_1^2+z_1z_2-\frac{n-1}2\frac{\delta_{\varDelta_\theta}}{\varepsilon_{\varDelta_\theta}}z_1^2-\frac12\frac1{\varepsilon_{\bar\psi}}z_1^2-\\&(l_\theta-\hat\theta)^{\mathrm T}(\varGamma^{-1}\dot{\hat\theta}-\tau_1)\end{aligned} \tag{11}$$

Step 2: Differentiating z_2 yields

$$\begin{aligned}\dot z_2=\;&\dot x_2-\dot\alpha_1=\\&z_3+\alpha_2+\omega_2^{\mathrm T}\hat\theta+\omega_2^{\mathrm T}(l_\theta-\hat\theta)+\omega_2^{\mathrm T}\varDelta_\theta-\\&\frac{\partial\alpha_1}{\partial x_1}x_2-\frac{\partial\alpha_1}{\partial\hat\theta}\dot{\hat\theta}\end{aligned} \tag{12}$$

where $\omega_2=\phi_2-\dfrac{\partial\alpha_1}{\partial x_1}\phi_1$.

Define the Lyapunov function V_2 as

$$V_2=V_1+\frac12z_2^2 \tag{13}$$

Taking the derivative of V_1, we get

$$\begin{aligned}\dot V_2=\;&\dot V_1+z_2\dot z_2\leqslant\\&-c_1z_1^2+z_2z_3+z_2\left[z_1+\alpha_2+\omega_2^{\mathrm T}\hat\theta-\frac{\partial\alpha_1}{\partial x_1}x_2-\right.\\&\left.\frac{\partial\alpha_1}{\partial\hat\theta}\dot{\hat\theta}\right]+z_2\omega_2^{\mathrm T}\varDelta_\theta-\frac{n-1}2\frac{\delta_{\varDelta_\theta}}{\varepsilon_{\varDelta_\theta}}z_1^2-\frac12\frac1{\varepsilon_{\bar\psi}}z_1^2-\\&(l_\theta-\hat\theta)^{\mathrm T}\left(\varGamma^{-1}\dot{\hat\theta}-\tau_1-\omega_2z_2\right)\end{aligned} \tag{14}$$

Similarly, we can derive

$$z_2\omega_2^{\mathrm T}\varDelta_\theta=z_2\varDelta_\theta^{\mathrm T}\bar W_2\underline{z}_2\leqslant\frac12\left(\frac{\delta_{\varDelta_\theta}}{\varepsilon_{\varDelta_\theta}}+\delta_{\varDelta_\theta}\varepsilon_{\varDelta_\theta}\|\bar W_2\|_{\mathrm F}^2\right)z_2^2+\frac12\frac{\delta_{\varDelta_\theta}}{\varepsilon_{\varDelta_\theta}}z_1^2 \tag{15}$$

Define the tuning function τ_2 and the virtual control law α_2 as

$$\tau_2=\ \tau_1+\omega_2z_2 \tag{16}$$

$$\begin{aligned}\alpha_2=&-c_2z_2-z_1-\omega_2^{\mathrm T}\hat\theta+\frac{\partial\alpha_1}{\partial x_1}x_2+\frac{\partial\alpha_1}{\partial\hat\theta}\varGamma\tau_2-\\&\frac{n-1}2\frac{\delta_{\varDelta_\theta}}{\varepsilon_{\varDelta_\theta}}z_2-\frac12\delta_{\varDelta_\theta}\varepsilon_{\varDelta_\theta}\|\bar W_2\|_{\mathrm F}^2z_2-\frac12\frac1{\varepsilon_{\bar\psi}}z_2\end{aligned} \tag{17}$$

where $c_2>0$.

Substituting Eqs. (16) and (17) into Formula (14), we get

$$\begin{aligned}\dot V_2\leqslant&-c_1z_1^2-c_2z_2^2+z_2z_3+z_2\frac{\partial\alpha_1}{\partial\hat\theta}\left(\varGamma\tau_2-\dot{\hat\theta}\right)-\\&\frac{n-2}2\frac{\delta_{\varDelta_\theta}}{\varepsilon_{\varDelta_\theta}}\|\underline{z}_2\|^2-(l_\theta-\hat\theta)^{\mathrm T}\left(\varGamma^{-1}\dot{\hat\theta}-\tau_2\right)-\\&\frac12\frac1{\varepsilon_{\bar\psi}}\|\underline{z}_2\|^2\end{aligned} \tag{18}$$

Step i ($i=3,4,\ldots,n-1$): Building upon the preceding two steps, we can derive recursively that

$$\begin{aligned}\dot z_i=\;&\dot x_i-\dot\alpha_{i-1}=\\&z_{i+1}+\alpha_i+\omega_i^{\mathrm T}\hat\theta+\omega_i^{\mathrm T}(l_\theta-\hat\theta)+\omega_i^{\mathrm T}\varDelta_\theta-\\&\sum_{j=1}^{i-1}\frac{\partial\alpha_{i-1}}{\partial x_j}x_{j+1}-\frac{\partial\alpha_{i-1}}{\partial\hat\theta}\dot{\hat\theta}\end{aligned} \tag{19}$$

where $\omega_i=\phi_i-\sum\limits_{j=1}^{i-1}\dfrac{\partial\alpha_{i-1}}{\partial x_j}\phi_j$.

Define the Lyapunov function V_i as

$$V_i = V_{i-1} + \frac{1}{2} z_i^2 \tag{20}$$

Taking the derivative of V_i, we get

$$\begin{aligned}\dot{V}_i = \dot{V}_{i-1} + z_i \dot{z}_i \leqslant & \\ & -\sum_{k=1}^{i-1} c_k z_k^2 + z_i z_{i+1} + z_i \Bigg(z_{i-1} + \alpha_i + \omega_i^{\mathrm{T}} \hat{\theta} - \\ & \sum_{j=1}^{i-1} \frac{\partial \alpha_{i-1}}{\partial x_j} x_{j+1} - \frac{\partial \alpha_{i-1}}{\partial \hat{\theta}} \dot{\hat{\theta}} \Bigg) + z_i \omega_i^{\mathrm{T}} \varDelta_\theta - \\ & (l_\theta - \hat{\theta})^{\mathrm{T}} \left(\varGamma^{-1} \dot{\hat{\theta}} - \tau_{i-1} - \omega_i z_i \right) + \\ & \left(\sum_{j=2}^{i-1} z_j \frac{\partial \alpha_{j-1}}{\partial \hat{\theta}} \right) (\varGamma \tau_{i-1} - \dot{\hat{\theta}}) - \\ & \frac{n-i+1}{2} \frac{\delta_{\varDelta_\theta}}{\varepsilon_{\varDelta_\theta}} \| \underline{z}_{i-1} \|^2 - \frac{1}{2} \frac{1}{\varepsilon_{\bar{\psi}}} \| \underline{z}_{i-1} \|^2 \end{aligned} \tag{21}$$

Similarly, we can derive

$$\begin{aligned} z_i \omega_i^{\mathrm{T}} \varDelta_\theta = z_i \varDelta_\theta^{\mathrm{T}} \bar{W}_i \underline{z}_i \leqslant & \\ & \frac{1}{2} \left(\frac{\delta_{\varDelta_\theta}}{\varepsilon_{\varDelta_\theta}} + \delta_{\varDelta_\theta} \varepsilon_{\varDelta_\theta} \| \bar{W}_i \|_{\mathrm{F}}^2 \right) z_i^2 + \frac{1}{2} \frac{\delta_{\varDelta_\theta}}{\varepsilon_{\varDelta_\theta}} \| \underline{z}_{i-1} \|^2 \end{aligned} \tag{22}$$

Define the tuning function τ_i and the virtual control law α_i as

$$\tau_i = \tau_{i-1} + \omega_i z_i = \sum_{j=1}^{i} \omega_j z_j \tag{23}$$

$$\begin{aligned} \alpha_i = & -c_i z_i - z_{i-1} - \omega_i^{\mathrm{T}} \hat{\theta} + \sum_{j=1}^{i-1} \frac{\partial \alpha_{i-1}}{\partial x_j} x_{j+1} + \\ & \frac{\partial \alpha_{i-1}}{\partial \hat{\theta}} \varGamma \tau_i + \sum_{j=2}^{i-1} \frac{\partial \alpha_{j-1}}{\partial \hat{\theta}} \varGamma \omega_i z_j - \frac{1}{2} \frac{1}{\varepsilon_{\bar{\psi}}} z_i - \\ & \frac{n-i+1}{2} \frac{\delta_{\varDelta_\theta}}{\varepsilon_{\varDelta_\theta}} z_i - \frac{1}{2} \delta_{\varDelta_\theta} \varepsilon_{\varDelta_\theta} \| \bar{W}_i \|_{\mathrm{F}}^2 z_i \end{aligned} \tag{24}$$

where $c_i > 0$.

Substituting Eqs. (23) and (24) into Formula (21), we get

$$\begin{aligned} \dot{V}_i \leqslant & -\sum_{k=1}^{i} c_k z_k^2 + z_i z_{i+1} + \left(\sum_{j=2}^{i} z_j \frac{\partial \alpha_{j-1}}{\partial \hat{\theta}} \right) (\varGamma \tau_i - \dot{\hat{\theta}}) - \\ & (l_\theta - \hat{\theta})^{\mathrm{T}} \left(\varGamma^{-1} \dot{\hat{\theta}} - \tau_i \right) \frac{n-i}{2} \frac{\delta_{\varDelta_\theta}}{\varepsilon_{\varDelta_\theta}} \| \underline{z}_i \|^2 - \\ & \frac{1}{2} \frac{1}{\varepsilon_{\bar{\psi}}} \| \underline{z}_i \|^2 \end{aligned} \tag{25}$$

Step *n*: Differentiating z_n yields

$$\begin{aligned} \dot{z}_n = \dot{x}_n - \dot{\alpha}_{n-1} = & \\ & \omega_n^{\mathrm{T}} \hat{\theta} + \omega_n^{\mathrm{T}} (l_\theta - \hat{\theta}) + \omega_n^{\mathrm{T}} \varDelta_\theta + bu - \\ & \sum_{j=1}^{n-1} \frac{\partial \alpha_{n-1}}{\partial x_j} x_{j+1} - \frac{\partial \alpha_{n-1}}{\partial \hat{\theta}} \dot{\hat{\theta}} \end{aligned} \tag{26}$$

where $\omega_n = \phi_n - \sum_{j=1}^{n-1} \frac{\partial \alpha_{n-1}}{\partial x_j} \phi_j$.

Define the Lyapunov function V as

$$V = V_{n-1} + \frac{1}{2} z_n^2 + \frac{|l_b|}{2 r_\varrho} \left(\frac{1}{l_b} - \hat{\varrho} \right)^2 \tag{27}$$

where l_b satisfies Assumption 2, $\hat{\varrho}$ is an estimate of $1/l_b$, and $r_\varrho > 0$ is a constant.

Remark 2 The core idea of the variable congelation method lies in using the congealed parameters l_θ and l_b to replace the time-varying parameters $\theta(t)$ and $b(t)$ in the design of the Lyapunov function. The selection of l_θ and l_b can be flexibly determined according to specific application requirements.

Taking the derivative of V, we get

$$\begin{aligned} \dot{V} = \dot{V}_{n-1} + z_n \dot{z}_n - \frac{|l_b|}{r_\varrho} \left(\frac{1}{l_b} - \hat{\varrho} \right) \dot{\hat{\varrho}} \leqslant & \\ & -\sum_{k=1}^{n-1} c_k z_k^2 + z_n \Bigg(z_{n-1} + bu + \omega_n^{\mathrm{T}} \hat{\theta} - \frac{\partial \alpha_{n-1}}{\partial \hat{\theta}} \dot{\hat{\theta}} - \\ & \sum_{j=1}^{n-1} \frac{\partial \alpha_{n-1}}{\partial x_j} x_{j+1} \Bigg) + z_n \omega_n^{\mathrm{T}} \varDelta_\theta - \frac{|l_b|}{r_\varrho} \left(\frac{1}{l_b} - \hat{\varrho} \right) \dot{\hat{\varrho}} - \\ & (l_\theta - \hat{\theta})^{\mathrm{T}} \left(\varGamma^{-1} \dot{\hat{\theta}} - \tau_{n-1} - \omega_n z_n \right) + \\ & \left(\sum_{j=2}^{n-1} z_j \frac{\partial \alpha_{j-1}}{\partial \hat{\theta}} \right) (\varGamma \tau_{n-1} - \dot{\hat{\theta}}) - \\ & \frac{1}{2} \frac{\delta_{\varDelta_\theta}}{\varepsilon_{\varDelta_\theta}} \| \underline{z}_{n-1} \|^2 - \frac{1}{2} \frac{1}{\varepsilon_{\bar{\psi}}} \| \underline{z}_{n-1} \|^2 \end{aligned} \tag{28}$$

Likewise, we can derive

$$\begin{aligned} z_n \omega_n^{\mathrm{T}} \varDelta_\theta = z_n \varDelta_\theta^{\mathrm{T}} \bar{W}_n \underline{z}_n \leqslant & \\ & \frac{1}{2} \left(\frac{\delta_{\varDelta_\theta}}{\varepsilon_{\varDelta_\theta}} + \delta_{\varDelta_\theta} \varepsilon_{\varDelta_\theta} \| \bar{W}_n \|_{\mathrm{F}}^2 \right) z_n^2 + \\ & \frac{1}{2} \frac{\delta_{\varDelta_\theta}}{\varepsilon_{\varDelta_\theta}} \| \underline{z}_{n-1} \|^2 \end{aligned} \tag{29}$$

Define the tuning function τ_n as

$$\tau_n = \sum_{j=1}^{n} \omega_j z_j \tag{30}$$

To further conserve communication resources, a DETM is designed as

$$u(t) = u_e(t_k),\ t \in [t_k, t_k + 1) \tag{31}$$

$$t_{k+1} = \inf \left\{ t \in \mathbb{R} : |u(t) - u_e(t)| \geqslant b_l + \frac{1}{a_l} \xi \right\} \tag{32}$$

where t_k is the event-triggering instant and $u_e(t)$ is the designed input. In the time interval $t \in [t_k, t_k + 1)$, the output is still $u_e(t_k)$. When Eq. (32) is satisfied, the time is updated to t_{k+1}.

It is evident that there exists a $\gamma > 0$ such that $|u(t) - u_e(t)| \leqslant \gamma$. a_l and b_l are positive constants. The dynamic threshold parameter ξ is defined by

$$\dot{\xi} = -\rho_l \xi + a_l (b_l - |u(t) - u_e(t)|) \tag{33}$$

From the above, we derive

$$
\begin{aligned}
\dot{V} \leqslant & -\sum_{k=1}^{n-1} c_k z_k^2 + z_n\Big(z_{n-1} + \omega_n^{\mathrm{T}}\hat{\theta} - \sum_{j=1}^{n-1}\frac{\partial \alpha_{n-1}}{\partial x_j}x_{j+1} - \\
& \frac{\partial \alpha_{n-1}}{\partial \hat{\theta}}\dot{\hat{\theta}} - \sum_{j=2}^{n-1}\frac{\partial \alpha_{j-1}}{\partial \hat{\theta}}\Gamma\omega_n z_j\Big) - \frac{1}{2}\frac{1}{\varepsilon_{\bar{\psi}}}\|\underline{z}_{n-1}\|^2 + \\
& \frac{1}{2}\left(\frac{\delta_{\varDelta_\theta}}{\varepsilon_{\varDelta_\theta}} + \delta_{\varDelta_\theta}\varepsilon_{\varDelta_\theta}\|\bar{W}_n\|_{\mathrm{F}}^2\right)z_n^2 - \frac{|l_b|}{r_\varrho}\Big(\frac{1}{l_b} - \hat{\varrho}\Big)\dot{\hat{\varrho}} - \\
& (l_\theta - \hat{\theta})^{\mathrm{T}}\left(\Gamma^{-1}\dot{\hat{\theta}} - \tau_n\right) + b(u - u_e)z_n + \\
& b u_e z_n
\end{aligned}
\tag{34}
$$

By applying Lemma 1, Formula (35) is obtained

$$
b(u-u_e)z_n \leqslant |b|\gamma|z_n| \leqslant \bar{b}\gamma\left(\frac{{z_n}^2}{\sqrt{{z_n}^2+\sigma^2}}+\sigma\right) \tag{35}
$$

where $\sigma = \mathrm{e}^{-\eta t}$, $\eta > 0$ is a constant.

Design $\varDelta_b = b(t) - l_b$ and $u_e = \hat{\varrho}\bar{u}$, then $b(t)$ can be decomposed into

$$
b u_e z_n = z_n\bar{u} - l_b\left(\frac{1}{l_b} - \hat{\varrho}\right)\bar{u}z_n + \varDelta_b\hat{\varrho}\bar{u}z_n \tag{36}
$$

From the above, we derive

$$
\begin{aligned}
\dot{V} \leqslant & \sum_{k=1}^{n-1} c_k z_k^2 + z_n\Big(z_{n-1} + \omega_n^{\mathrm{T}}\hat{\theta} - \sum_{j=1}^{n-1}\frac{\partial \alpha_{n-1}}{\partial x_j}x_{j+1} - \\
& \frac{\partial \alpha_{n-1}}{\partial \hat{\theta}}\dot{\hat{\theta}} - \sum_{j=2}^{n-1}\frac{\partial \alpha_{j-1}}{\partial \hat{\theta}}\Gamma\omega_n z_j + \frac{\bar{b}\gamma z_n}{\sqrt{{z_n}^2+\sigma^2}}\Big) + \\
& \bar{b}\gamma\sigma + z_n\bar{u} + \frac{1}{2}\Big(\frac{\delta_{\varDelta_\theta}}{\varepsilon_{\varDelta_\theta}} + \delta_{\varDelta_\theta}\varepsilon_{\varDelta_\theta}\|\bar{W}_n\|_{\mathrm{F}}^2\Big)z_n^2 - \\
& \frac{1}{2}\frac{1}{\varepsilon_{\bar{\psi}}}\|\underline{z}_{n-1}\|^2 - (l_\theta - \hat{\theta})^{\mathrm{T}}\left(\Gamma^{-1}\dot{\hat{\theta}} - \tau_n\right) + \\
& \Big(\sum_{j=2}^{n-1} z_j\frac{\partial \alpha_{j-1}}{\partial \hat{\theta}}\Big)(\Gamma\tau_n - \dot{\hat{\theta}}) + \varDelta_b\hat{\varrho}\bar{u}z_n - \\
& l_b\left(\frac{1}{l_b} - \hat{\varrho}\right)\bar{u}z_n - \frac{|l_b|}{r_\varrho}\left(\frac{1}{l_b} - \hat{\varrho}\right)\dot{\hat{\varrho}}
\end{aligned}
\tag{37}
$$

Design the parameter adaptive law $\dot{\hat{\theta}}$ as

$$
\dot{\hat{\theta}} = \Gamma\tau_n = \Gamma\sum_{j=1}^{n}\omega_j z_j \tag{38}
$$

Then, we have

$$
\begin{aligned}
\dot{V} \leqslant & \sum_{k=1}^{n-1} c_k z_k^2 + z_n\psi + \bar{b}\gamma\sigma + z_n\bar{u} + \varDelta_b\hat{\varrho}\bar{u}z_n + \\
& \frac{1}{2}\left(\frac{\delta_{\varDelta_\theta}}{\varepsilon_{\varDelta_\theta}} + \delta_{\varDelta_\theta}\varepsilon_{\varDelta_\theta}\|\bar{W}_n\|_{\mathrm{F}}^2\right)z_n^2 - \frac{1}{2}\frac{1}{\varepsilon_{\bar{\psi}}}\|\underline{z}_{n-1}\|^2 - \\
& l_b\left(\frac{1}{l_b} - \hat{\varrho}\right)\bar{u}z_n - \frac{|l_b|}{r_\varrho}\left(\frac{1}{l_b} - \hat{\varrho}\right)\dot{\hat{\varrho}}
\end{aligned}
\tag{39}
$$

where

$$
\begin{aligned}
\psi = & z_{n-1} + \omega_n^{\mathrm{T}}\hat{\theta} - \sum_{j=1}^{n-1}\frac{\partial \alpha_{n-1}}{\partial x_j}x_{j+1} - \frac{\partial \alpha_{n-1}}{\partial \hat{\theta}}\Gamma\tau_n - \\
& \sum_{j=2}^{n-1}\frac{\partial \alpha_{j-1}}{\partial \hat{\theta}}\Gamma\omega_n z_j + \frac{\bar{b}\gamma z_n}{\sqrt{{z_n}^2+\sigma^2}}.
\end{aligned}
$$

By utilizing the Hadamard's lemma, we can derive

$$
\begin{aligned}
& z_n\psi = z_n\bar{\psi}^{\mathrm{T}}z \leqslant \\
& \frac{1}{2}\left(\frac{1}{\varepsilon_{\bar{\psi}}} + \varepsilon_{\bar{\psi}}\|\bar{\psi}\|^2\right)z_n^2 + \frac{1}{2}\frac{1}{\varepsilon_{\bar{\psi}}}\|\underline{z}_{n-1}\|^2
\end{aligned}
\tag{40}
$$

where $\bar{\psi}$ represents a smooth mapping, and $\varepsilon_{\bar{\psi}} > 0$ is a constant.

Design the control input $\bar{u}$ as

$$
\begin{aligned}
\bar{u} = & -c_n z_n - \frac{1}{2}\left(\frac{\delta_{\varDelta_\theta}}{\varepsilon_{\varDelta_\theta}} + \delta_{\varDelta_\theta}\varepsilon_{\varDelta_\theta}\|\bar{W}_n\|_{\mathrm{F}}^2\right)z_n - \\
& \frac{1}{2}\left(\frac{1}{\varepsilon_{\bar{\psi}}} + \varepsilon_{\bar{\psi}}\|\bar{\psi}\|^2\right)z_n = \\
& -Kz_n
\end{aligned}
\tag{41}
$$

where

$$
K = c_n + \frac{1}{2}\left(\frac{\delta_{\varDelta_\theta}}{\varepsilon_{\varDelta_\theta}} + \delta_{\varDelta_\theta}\varepsilon_{\varDelta_\theta}\|\bar{W}_n\|_{\mathrm{F}}^2\right) + \frac{1}{2}\left(\frac{1}{\varepsilon_{\bar{\psi}}} + \varepsilon_{\bar{\psi}}\|\bar{\psi}\|^2\right) > 0.
$$

From Eqs. (10), (16), (23), and (30), the parameter update law $\dot{\hat{\varrho}}$ can be designed as

$$
\begin{aligned}
& \dot{\hat{\varrho}} = -r_\varrho \mathrm{sgn}(l_b)\bar{u}z_n = \\
& r_\varrho \mathrm{sgn}(l_b)K{z_n}^2
\end{aligned}
\tag{42}
$$

In summary, we obtain

$$
\dot{V} \leqslant -\sum_{j=1}^{n} c_j z_j^2 + \varDelta_b\hat{\varrho}\bar{u}z_n + \bar{b}\gamma\sigma \tag{43}
$$

From Eq. (42) and Assumption 2, it can be observed that when $b(t) > 0$, for all $t \geqslant 0$, there exists a constant l_b such that $0 < l_b \leqslant b(t)$, so we can get $\varDelta_b > 0$ and $\dot{\hat{\varrho}} \geqslant 0$. Therefore, for all $t \geqslant 0$, any initial value $\hat{\varrho}(0) > 0$ can ensure that $\hat{\varrho}(t) > 0$. Analogously, when $b(t) < 0$, for all $t \geqslant 0$, there exists a constant l_b, such that $b(t) \leqslant l_b < 0$, so we get $\varDelta_b < 0$ and $\dot{\hat{\varrho}} \leqslant 0$. Therefore, for all $t \geqslant 0$, any initial value $\hat{\varrho}(0) < 0$ can ensure that $\hat{\varrho}(t) < 0$. Therefore, $\varDelta_b\hat{\varrho}\bar{u}z_n = -\varDelta_b\hat{\varrho}K{z_n}^2 < 0$.

Reviewing Formula (43), combined with $\varDelta_b\hat{\varrho}\bar{u}z_n < 0$, we have

$$
\begin{aligned}
\dot{V} \leqslant & -\sum_{j=1}^{n} c_j z_j^2 + \varDelta_b\hat{\varrho}\bar{u}z_n + \bar{b}\gamma\sigma \leqslant \\
& -\sum_{j=1}^{n} c_j z_j^2 + \bar{b}\gamma\sigma \leqslant \\
& -cV_z + \bar{b}\gamma\sigma
\end{aligned}
\tag{44}
$$

where $c = 2\min\{c_1, c_2, \ldots, c_n\}$ and $V_z = \frac{1}{2}\sum_{j=1}^{n}{z_j}^2$.

IV. Stability Analysis

Theorem 1 Consider a closed-loop system consisting of the system in Eq. (1) that satisfies Assumptions 1 and 2, and an event-trigger based adaptive controller designed in Formulas (31), (32), (38), (41), and (42). The system is globally uniformly asymptotically stable, that is, all signals of the closed-loop system are bounded and converge to 0.

Proof We know that

$$V = \frac{1}{2}\sum_{j=1}^{n} {z_j}^2 + \frac{1}{2}(l_\theta - \hat{\theta})^{\mathrm{T}} \Gamma^{-1}(l_\theta - \hat{\theta}) + \frac{|l_b|}{2r_\varrho}(\frac{1}{l_b} - \hat{\varrho})^2 = V_z + V_\theta + V_\rho \tag{45}$$

where

$$V_z = \frac{1}{2}\sum_{j=1}^{n} {z_j}^2,$$
$$V_\theta = \frac{1}{2}(l_\theta - \hat{\theta})^{\mathrm{T}} \Gamma^{-1}(l_\theta - \hat{\theta}),$$
$$V_\rho = \frac{|l_b|}{2r_\varrho}\left(\frac{1}{l_b} - \hat{\varrho}\right)^2.$$

Firstly, from Formulas (31) and (35), we have $|u(t) - u_e(t)| \leqslant \gamma$ and

$$\int_0^\infty \bar{b}\gamma\sigma \mathrm{d}t = \int_0^\infty \bar{b}\gamma \mathrm{e}^{-\xi t}\mathrm{d}t = \frac{\bar{b}\gamma}{\xi} < \infty \tag{46}$$

Formula (47) is obtained by integrating Formula (44) over the interval $[0, t)$

$$V(t) \leqslant V(0) - \int_0^t cV_z \mathrm{d}v + \int_0^t \bar{b}\gamma\sigma \mathrm{d}v \leqslant \infty \tag{47}$$

Then, we can get

$$\int_0^t cV_z \mathrm{d}v \leqslant V(0) + \int_0^t \bar{b}\gamma\sigma \mathrm{d}v < \infty \tag{48}$$

From Formula (47) we know that for $\forall t \in [0, \infty)$, $V(t)$ is bounded, so we get that V_z, V_θ, and V_ρ are also bounded, and that $z_i \ (i = 1, 2, \ldots, n)$, $\hat{\theta}$, and $\hat{\rho}$ are also bounded by expressions for V_z, V_θ, and V_ρ. In addition, according to Formula (48) and $z_i \in \mathcal{L}_2 \cap \mathcal{L}_\infty$, using the Barbalat's lemma, $\lim_{t\to\infty} z_i = 0$ can be obtained. $\lim_{t\to\infty} x_1 = 0$ can be obtained from Eq. (3). From the definition of α_1 in Eq. (9), we get $\lim_{t\to\infty} \alpha_1 = 0$. Further, since $x_2 = z_2 + \alpha_1$, $\lim_{t\to\infty} x_2 = 0$ is obtained. Analogously, we can recursively get $\lim_{t\to\infty} \alpha_i = 0$ and $\lim_{t\to\infty} x_i = 0$. Equation (41) shows that $\bar{u}$ is bounded and has $\lim_{t\to\infty} \bar{u} = 0$. Since $\hat{\rho} \in \mathcal{L}_\infty$ and $u_e = \hat{\rho}\bar{u}$, we have $u_e \in \mathcal{L}_\infty$. Finally, the DETM in Eq. (31) guarantees that u is consistently bounded. ■

Remark 3 The Barbalat's lemma is a commonly used tool in the stability analysis of nonlinear control systems. It is mainly used to prove that the derivative of a signal or a function converges to zero, thereby indicating the convergence of the system state.

Remark 4 This paper is inspired by Chen and Astolfi in dealing with time-varying parameters [18]. The difference is that this paper incorporates a DETM, and the error asymptotically converges to 0, which achieves better convergence performance than bounded stability.

V. Simulation

Give the controlled mass-spring model

$$m\ddot{d} + F_1 + F_2 = F \tag{49}$$

where m represents the mass of the slider. The displacement of d is measured relative to the reference position. $F_1 = c(1 + \sin t)v$ represents a frictional resistance, where $v = \dot{d}$ is the sliding velocity and $c > 0$ is an unknown constant. $F_2 = kd$ represents the restoring force of the spring, where k is the spring stiffness coefficient, which is an unknown constant. $F = c(1 + 0.5\sin t)u$ represents a time-varying external force acting on the slider.

Let $x_1 = y$ and $x_2 = \dot{y}$. Then, Eq. (49) can be written as

$$\begin{aligned} \dot{x}_1 &= x_2, \\ \dot{x}_2 &= \frac{c(1+0.5\sin t)}{m}u - \frac{k}{m}x_1 - \frac{c(1+\sin t)}{m}x_2 \end{aligned} \tag{50}$$

It is evident that Eq. (50) fulfills Assumptions 1 and 2 with $\theta(t) = [\theta_{21}, \theta_{22}]^{\mathrm{T}} = \left[\frac{k}{m}, \frac{c(1+\sin t)}{m}\right]^{\mathrm{T}}$, $\phi_1(x_1) = 0$, $\phi_2(x_2) = [-x_1, -x_2]$, and $b(t) = \frac{c(1+0.5\sin t)}{m}$.

Let $m = k = c = 1$, $c_1 = 0.06$, $c_2 = 0.1$, $\Gamma = \begin{bmatrix} \Gamma_1 & 0 \\ 0 & \Gamma_2 \end{bmatrix}$, $\Gamma_1 = 0.2$, $\Gamma_2 = 0.3$, $l_b = 0.4$, $r_\varrho = 0.23$, $\varepsilon_{\bar{\psi}} = 0.2$, $\delta_{\Delta_\theta} = 1.1839$, and $\varepsilon_{\Delta_\theta} = 0.8$. Assume the initial values $x_1(0) = 1$, $x_2(0) = -0.6$, $\hat{\theta}_{21}(0) = 1.1$, $\hat{\theta}_{22}(0) = 0$, $\hat{\varrho}(0) = 0.1$, and $\xi(0) = 0$. The relevant parameters are $a_l = 0.8$, $b_l = 0.01$, $\varrho_l = 0.02$, $\Gamma_1 = 0.2$, and $\Gamma_2 = 0.3$. Simulation outcomes are displayed in Figs. 1–4.

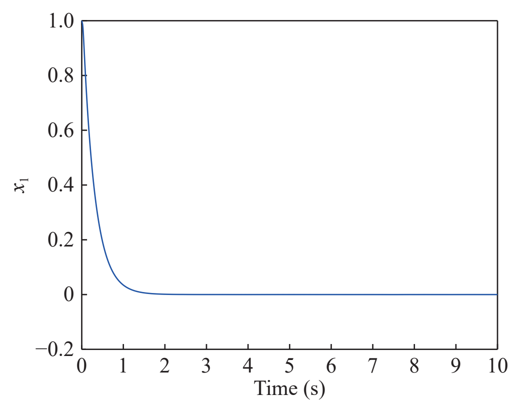

Figure 1 System state x_1.

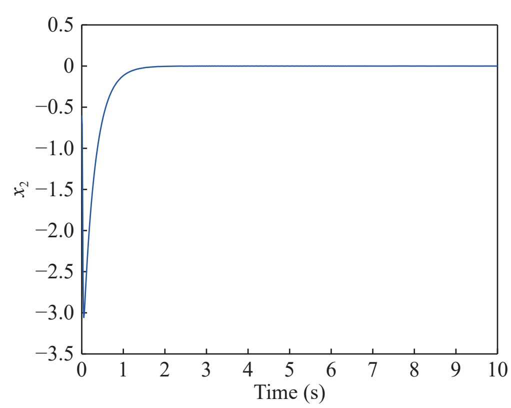

Figure 2 System state x_2.

Figures 1 and 2 display the simulation results of system states x_1 and x_2, respectively. It can be observed that under the designed controller, both state variables asymptotically converge to zero, demonstrating the effectiveness of the control strategy. Figure 3 shows the control input trajectory, while Fig. 4 presents the event-triggering intervals under the proposed DETM.

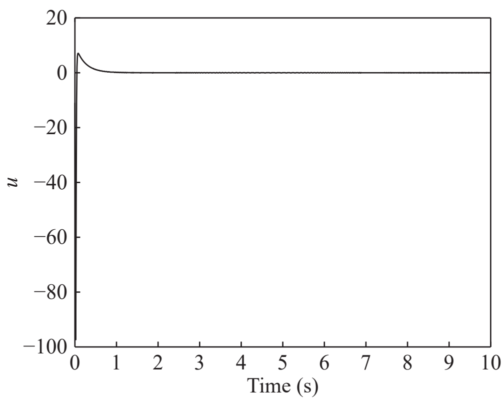

Figure 3 Input u.

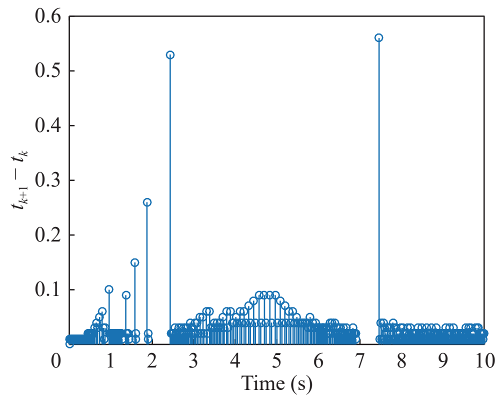

Figure 4 Time interval of DETM.

To further evaluate the performance of the proposed DETM scheme, a comparative simulation with the static ETM in Ref. [29] is conducted using the same controller parameters and different initial conditions. Figure 5 illustrates the triggering intervals under static ETM. In addition, Table 1 summarizes the numbers of triggering events for both methods. It is evident that DETM significantly reduces the number of triggers while maintaining satisfactory control performance.

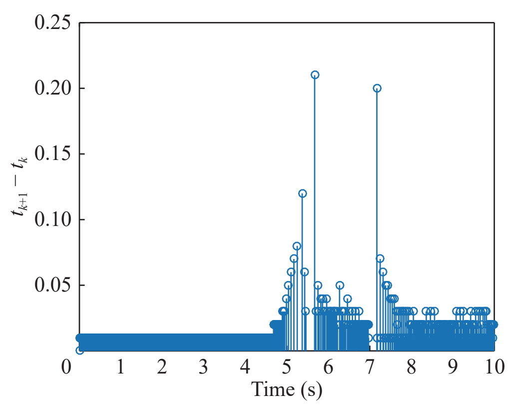

Figure 5 Time interval of static ETM.

Table 1 Number of transmission events of static ETM and DETM.

Control strategy	Number of triggers
Static ETM	695
DETM	362

VI. CONCLUSION

This article introduces an adaptive asymptotic dynamic ETC strategy for strict-feedback NTS. The congelation of variables approach is utilized to manage the time-varying parameters in the system. The controller is constructed by employing the backstepping approach. To enhance the efficiency of communication resources, a DETM is incorporated into the control procedure. The stability of this system is demonstrated via the Lyapunov stability theorem analysis. Furthermore, the simulation outcomes verify the practicality and effectiveness.

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