Edge-Event-Triggered Sliding Mode Consensus Control of Heterogeneous Multi-Agent System with Unknown Disturbance

Shuangsi Xue, Huan Li, Junkai Tan, Zihang Guo, and Hui Cao

***Abstract*—This paper proposes a novel edge-event-triggered sliding mode control strategy to achieve consensus of heterogeneous multi-agent systems (MASs) consisting of the first-order and second-order agents under the influence of unknown disturbances. The edge-event-triggered mechanism (EETM) is uniquely defined and differs from traditional node-based approaches by focusing on individual edges rather than nodes, resulting in more efficient communication. The edge-event-triggered mechanism utilizes the relative positions between an agent and its neighbors, independent of global coordinates. The designed controller has two components. First, the sliding mode control strategy enables heterogeneous MASs to reach and maintain their states on the sliding mode surface despite unknown disturbances. Subsequently, the equivalent control strategy, combined with the edge-event-triggered mechanism, guarantees the consensus of heterogeneous MASs while avoiding Zeno behavior. Triggering functions are developed for agents that form the same edge, allowing asynchronous mechanisms triggered by events of the edge. By employing the Lyapunov stability theory, sufficient conditions for the proposed control strategy are given. Simulation results demonstrate the effectiveness of the theoretical results, showing significant communication reduction compared to traditional event-triggered approaches.**

***Index Terms*—Heterogeneous multi-agent system, consensus control, sliding mode, edge-event-triggered control, disturbance**

I. INTRODUCTION

THE control of multi-agent systems (MASs) has garnered significant interest in various fields, including robotics, sensor networks, and smart grids, due to their potential for coordinated tasks. Consensus control, which aims to synchronize the states of multiple agents to a common value through distributed algorithms, has been extensively investigated [1–6]. However, while considerable progress has been made in homogeneous MASs, the control of heterogeneous MASs presents unique challenges due to the diverse properties of individual agents.

Heterogeneous MASs more accurately represent real-world scenarios where system components may have varying capabilities and complexities. Systems with different order dynamics are particularly important in practical applications, such as mixed ground and aerial vehicle coordination or collaborative control of robots with different actuation capabilities. Recent research has explored various aspects of heterogeneous MASs control, including output formation tracking [7], average tracking [8], and output consensus [9]. Much of this work focuses on the first-order heterogeneous MASs with different state dimensions but the same output dimension, where heterogeneity is manifested in state matrices and dimensions. However, real scenarios often involve heterogeneous MASs with mixed orders of dynamics. In Ref. [10], the consensus of the second-order heterogeneous MASs is studied. Nevertheless, there remains a significant need for effective control protocols for heterogeneous MASs with mixed first-order and second-order dynamics, especially in the presence of external disturbances and communication constraints.

Continuous information transmission among agents, required by many classical control methods, can be impractical or inefficient in real-world applications due to limited communication resources [11–14]. Event-triggered control (ETC) addresses this limitation by updating control inputs only when necessary, based on specific triggering conditions, rather than at fixed time intervals [15–20]. Various ETC strategies have been proposed, ranging from centralized to distributed approaches, including periodic ETC [16] and combined communication/controller update triggers [20]. However, many conventional methods employ neighbor-based transmission, requiring each triggered agent to communicate with all its neighbors, which can be inefficient in dense or large-scale networks due to redundant information exchange.

Reducing communication is crucial for energy saving, bandwidth conservation, and network scalability. It is also advantageous to design control laws that require no global information, meaning that the control strategy should be fully distributed. While secure ETC protocols have been designed [21], some may still require global information like Laplacian eigenvalues, hindering fully distributed implementation. Therefore, significant research effort has focused on fully distributed control protocols, indicating that each agent makes decisions based solely on local information without requiring centralized coordination [22, 23]. Although these approaches

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achieve distributed implementation, many still rely on neighbor-based information transmission at trigger instants.

To address these limitations, edge-event-triggered control (EETC) has been introduced, where triggering is based on the state of individual communication edges rather than nodes [24–30]. This approach allows agents to selectively communicate with specific neighbors only when needed, leading to more efficient, fully distributed control protocols that operate independently of global network topology information. EETC has been applied to leader-following consensus [24], output tracking [28], and average consensus [29].

Sliding mode control (SMC) has emerged as a popular control technique for MASs owing to its inherent robustness against uncertainties and external disturbances [31–34]. SMC enhances system performance by forcing the system trajectory onto a predefined sliding surface where the system becomes insensitive to matched disturbances and parameter variations [35–37]. Despite advances in applying SMC to MASs, including event-based SMC [35, 37] and decentralized SMC for heterogeneous MASs [36], the integration of SMC with edge-event-triggered mechanism (EETM) for heterogeneous systems remains relatively unexplored, particularly in addressing the trade-off between communication efficiency and disturbance rejection. Given the potential of SMC to enhance robustness, this paper utilizes integral SMC to tackle unknown disturbances in heterogeneous MASs under an EETC framework.

The primary motivation for this work stems from the practical need for robust consensus control in heterogeneous multi-agent systems characterized by agents with mixed first-order and second-order dynamics, which are prevalent in real-world applications like robotic team or vehicle coordination. Existing control strategies often struggle with the combined challenges of agent heterogeneity, unknown external disturbances, and the need for communication efficiency in resource-constrained networks. Traditional node-based event-triggered methods can lead to unnecessary communication overhead, while ensuring robustness against disturbances in heterogeneous systems requires specialized control techniques like SMC. This paper aims to bridge this gap by proposing a novel edge-event-triggered sliding mode control (EET-SMC) strategy.

This paper concentrates on heterogeneous MASs consisting of the first-order and second-order agents subject to unknown disturbances. To decrease resource consumption and implement the control protocol in a fully distributed manner, an edge-event-triggered mechanism is designed. Varying agent dynamics make designing triggering conditions challenging. Drawing inspiration from the above discussions, we propose an EET-SMC consensus strategy. The primary contributions of this paper are summarized as follows:

(1) A new edge-event-triggered mechanism is proposed, specifically designed for heterogeneous MASs comprising the first-order and second-order agents, enabling selective, asynchronous communication between connected agents to significantly conserve communication resources compared to node-based approaches.

(2) The proposed control strategy, leveraging the EETM and relative state information, is designed to be fully distributed, requiring no global information about the network topology or centralized coordination for its implementation.

(3) Integral sliding mode surfaces are designed for both first-order and second-order agents and are incorporated into the control protocol to ensure robustness against unknown matched disturbances and achieve finite-time convergence to the sliding manifold.

(4) The control law relies only on locally available relative position and velocity information, enhancing practical applicability in scenarios without global positioning. Rigorous analysis confirms consensus achievement and Zeno-free operation.

The rest of this paper is organized as follows. Some basic preliminaries and the problem formulation are provided in Section II. Section III proposes the edge-event-triggered sliding mode control protocol. Section IV presents the main theoretical results of the proposed control strategy. A numerical simulation example validates the feasibility of the proposed protocols in Section V. Finally, some conclusions are provided in Section VI.

Notation $\mathbb{R}$ denotes the set of real numbers. $\|\cdot\|$ represents the Euclidean norm (2-norm) for vectors, while $|\cdot|$ denotes the absolute value for scalars. $Q = \mathrm{diag}\{q_1, q_2, ..., q_N\}$ represents the diagonal matrix with diagonal elements $q_1, q_2, ..., q_N$. $D^+(\cdot)$ denotes the right-hand derivative of a function.

II. Problem Formulation and Preliminary

A. Graph Theory

The communication topology of N agents can be modeled by an undirected graph $G = (V, E, A)$. We use v_i to denote node i. The node set $V = \{v_1, v_2, ..., v_N\}$ represents the agents and the edge set $E \subseteq V \times V$ describes the interaction relationship of the MAS. For any two different agents, if an undirected path exists between them, the undirected graph is connected. The adjacency matrix is denoted by $A = [a_{ij}] \in \mathbb{R}^{N \times N}$. If an edge $(i, j) \in E$, then $a_{ij} > 0$, otherwise $a_{ij} = 0$. When the communication topology G is undirected, $a_{ij} = a_{ji}$, $\forall i, j \in V$. Due to the requirement of no self-edges, $a_{ii} = 0$. The neighbor set of the node v_i is defined as $N_i = \{j \in V | (j, i) \in E\}$. The in-degree of node i is represented by $d_i = \sum_{j=1}^{N} a_{ij}$, and we denote the in-degree matrix as $D = \mathrm{diag}\{d_1, d_2, ..., d_N\}$.

Assumption 1 The undirected graph $G = (V, E, A)$ is connected.

B. System Description

Consider a heterogeneous multi-agent system consisting of S second-order agents and $N - S$ ($N > S$) first-order agents. The dynamic is described by

$$\begin{cases}\dot{x}_i(t) = v_i(t), \\ \dot{v}_i(t) = g_i(x_i, v_i, t)u_i(t) + f_i(x_i, v_i, t) + \delta_i(t), \ i \in I_S\end{cases} \tag{1}$$

$$\dot{x}_i(t) = g_i(x_i, t)u_i(t) + f_i(x_i, t) + \delta_i(t), \ i \in I_{N-S} \tag{2}$$

where s_i is the time instant. $I_S = \{1, 2, ..., S\}$ and $I_{N-S} = \{S+1, S+2, ..., N\}$ represent the sets of second-order and first-order agents, respectively. $x_i(t)$, $v_i(t)$, and $u_i(t) \in \mathbb{R}$ are the position state, the velocity state, and the control input of agent i. $f_i(x_i, v_i, t)$ and $f_i(x_i, t) \in \mathbb{R}$ are the known nonlinear dynamics of agent i. The functions $g_i(x_i, v_i, t)$ and $g_i(x_i, t) \in \mathbb{R}$ represent control input gains that modulate the control effect. They are assumed to be non-zero with known signs (typically positive) to ensure controllability of the system, allowing for the existence of their inverse functions $g_i^{-1}(x_i, v_i, t)$ and $g_i^{-1}(x_i, t)$. $\delta_i(t) \in \mathbb{R}$ is the unknown disturbance affecting agent i.

Definition 1 The heterogeneous multi-agent system in Eqs. (1) and (2) is considered to achieve consensus if its position and velocity states satisfy the following conditions

$$\begin{cases}\lim\limits_{t\to\infty} |x_i(t) - x_j(t)| = 0, \ i, j \in I_N; \\ \lim\limits_{t\to\infty} |v_i(t)| = 0, \ i \in I_{N-S}; \\ \lim\limits_{t\to\infty} |v_i(t)| = 0, \ i \in I_S\end{cases} \tag{3}$$

Assumption 2 The unknown disturbance $\delta_i(t)$ affecting each agent i is bounded. Furthermore, it is assumed that an upper bound $\bar{\delta}$ exists such that $|\delta_i(t)| \leqslant \bar{\delta}$ for all $i \in I_N$ and $t \geqslant 0$, and this bound $\bar{\delta} > 0$ is known or can be conservatively estimated for the purpose of control design (specifically for choosing parameter σ in Eq. (11)).

Definition 2 Zeno behavior refers to the phenomenon where an infinite number of events occur within a finite time interval. In the context of event-triggered control, it describes a situation where the triggering condition is met infinitely often in a finite time period, making the implementation of the control strategy impractical.

Lemma 1 [38] Considering a system modeled by $\dot{x} = F(x)$ with $F(0) = 0$, $x(0) = x_0$, and $x \in \mathbb{R}^N$, assume that there exists a positive definite function $V(x)$ defined in the vicinity of the origin. The system is finite-time stable if $V(x)$ satisfies that

$$\dot{V}(x) + \beta V^{\gamma}(x) \leqslant 0 \tag{4}$$

where $\beta > 0$ and $0 < \gamma < 1$. The upper limit of the setting time T_s complies with

$$T_s \leqslant \frac{V(x_0)^{1-\gamma}}{\beta(1-\gamma)} \tag{5}$$

Lemma 2 [39] For $x_i \in \mathbb{R}$, $i = 1, 2, ..., n$, and $0 < r < 1$, then

$$\left(\sum_{i=1}^{n} |x_i|\right)^r \leqslant \sum_{i=1}^{n} |x_i|^r \tag{6}$$

III. Design of Edge-Event-Triggered Sliding Mode Control

This section presents the design of our novel edge-event-triggered sliding mode control strategy for heterogeneous multi-agent systems. We first introduce the control protocol, then design the integral sliding mode surfaces, and finally propose the edge-event-triggered mechanism.

A. Control Protocol Design

To enable control based on relative information rather than absolute positions, we define $\xi_{ij}(t) = x_i(t) - x_j(t)$, $i, j \in I_N$ as the relative position state difference between agents v_i and v_j. $\xi_{ij}(t_k^{ij})$ represents the sampling value of $\xi_{ij}(t)$ at time t_k^{ij}. $\xi_{ij}(t_k^{ij})$ is sampled by agent v_i and it is only available to v_i. The sampling value of $v_i(t)$ (for $i \in I_S$) at t_k^{ij} is denoted as $v_i(t_k^{ij})$. By using a zero-order holder (ZOH), the discrete signals $\xi_{ij}(t_k^{ij})$ and $v_i(t_k^{ij})$ are converted to continuous signals $\hat{\xi}_{ij}(t)$ and $\hat{v}_{ij}(t)$, which are defined as

$$\hat{\xi}_{ij}(t) = \xi_{ij}(t_k^{ij}), \ t \in [t_k^{ij}, t_{k+1}^{ij}) \tag{7}$$

$$\hat{v}_{ij}(t) = v_i(t_k^{ij}), \ t \in [t_k^{ij}, t_{k+1}^{ij}) \tag{8}$$

Our control protocol is designed with a two-component structure to address both the consensus objective and the disturbance rejection challenge. The control input $u_i(t)$ can be presented as

$$\begin{cases}u_i(t) = g_i^{-1}(x_i, v_i, t)(u_i^{\mathrm{eq}}(t) + u_i^s(t)), \ i \in I_S; \\ u_i(t) = g_i^{-1}(x_i, t)(u_i^{\mathrm{eq}}(t) + u_i^s(t)), \ i \in I_{N-S}\end{cases} \tag{9}$$

where $u_i^{\mathrm{eq}}(t)$ is the equivalent edge-event-triggered consensus control function and $u_i^s(t)$ is the sliding mode control function, which are given as

$$\begin{cases}u_i^{\mathrm{eq}}(t) = -\sum\limits_{j\in N_i} a_{ij}(\hat{\xi}_{ij}(t) + \dfrac{1}{d_i}\mu\hat{v}_{ij}(t)) - \\ \qquad f_i(x_i, v_i, t), \ i \in I_S; \\ u_i^{\mathrm{eq}}(t) = -\sum\limits_{j\in N_i} a_{ij}(\hat{\xi}_{ij}(t)) - f_i(x_i, t), \ i \in I_{N-S}\end{cases} \tag{10}$$

$$u_i^s(t) = -\varpi|s_i|^{\lambda}\mathrm{sgn}(s_i) - \sigma\mathrm{sgn}(s_i), \ i \in I_N \tag{11}$$

where s_i is the integral sliding-mode surface, μ, ϖ, λ, and $\sigma > 0$ are designed parameters, and $\mathrm{sgn}(x)$ is the sign function, which is commonly described by (for scalar $x \in \mathbb{R}$)

$$\mathrm{sgn}(x) = \begin{cases}1, \ x > 0; \\ 0, \ x = 0; \\ -1, \ x < 0\end{cases} \tag{12}$$

The rationale behind this control design is twofold. (1) The equivalent control $u_i^{\mathrm{eq}}(t)$ drives the system towards a consensus based on the relative positions and velocities of neighboring agents. (2) The sliding mode control $u_i^s(t)$ ensures robustness against unknown disturbance $\delta_i(t)$.

It should be noted that the inclusion of the sign function in the sliding mode control component may lead to chattering phenomena. This can be alleviated by replacing the sign function with continuous approximations such as the saturation function $\mathrm{sat}(s_i/\varepsilon)=\min(1,\max(-1,s_i/\varepsilon))$ or the hyperbolic tangent function $\tanh(s_i/\varepsilon)$, where $\varepsilon>0$ is a small constant that determines the boundary layer thickness around the sliding surface.

B. Integral Sliding Mode Surface Design

The sampling errors of relative position and velocity are denoted as

$$\varepsilon_{ij}(t)=\hat{\xi}_{ij}(t)-\xi_{ij}(t) \tag{13}$$

$$\eta_{ij}(t)=\hat{v}_{ij}(t)-v_i(t) \tag{14}$$

We design integral sliding mode surfaces, which offer several advantages over conventional sliding surfaces, including enhanced robustness against disturbances and reduced steady-state errors. These surfaces are described by

$$s_i(t)=v_i(t)-v_i(0)+\int_0^t\left[\sum_{j\in N_i}a_{ij}\left(\hat{\xi}_{ij}(t)+\frac{1}{d_i}\mu\hat{v}_{ij}(t)\right)\right]\mathrm{d}t,\ i\in I_S \tag{15}$$

$$s_i(t)=x_i(t)-x_i(0)+\int_0^t\sum_{j\in N_i}a_{ij}\hat{\xi}_{ij}(t)\mathrm{d}t,\ i\in I_{N-S} \tag{16}$$

Once the system states reach these sliding surfaces ($s_i(t)=0$), they become insensitive to matched disturbance $\delta_i(t)$, eliminating their influence on the system dynamics.

C. Edge-Event-Triggered Mechanism Design

The edge-event-triggered mechanism represents a practical application scenario where communication resources are limited, and agents need to selectively communicate with specific neighbors only when necessary. This is particularly relevant in applications such as large-scale sensor networks, drone swarms, or mobile robot teams operating with battery constraints.

The triggering time instant t_k^{ij} is determined by the edge-event-triggered mechanism

$$t_{k+1}^{ij}=\inf\{t>t_k^{ij}|h_{ij}(t)\geqslant 0\},\ i,j\in I_N \tag{17}$$

where $h_{ij}(t)$ represents the triggering function, which is designed as

$$h_{ij}(t)=\zeta\kappa\varepsilon_{ij}^2(t)+\zeta\frac{\mu}{d_i}\eta_{ij}^2(t)-\left(\frac{\mu}{d_i}-\frac{1}{\kappa}\right)v_i^2(t),\ i\in I_S \tag{18}$$

$$h_{ij}(t)=\iota\gamma\varepsilon_{ij}^2(t)-\left(\frac{2}{d_i}-\frac{1}{\gamma}\right)\phi_i^2(t),\ i\in I_{N-S} \tag{19}$$

where κ and γ are positive constants to be specified and ζ, $\iota\geqslant 1$ are constants to adjust the triggering frequency. $\phi_i(t)$ is given as

$$\phi_i(t)=\sum_{j=1}^{N}a_{ij}\xi_{ij}(t) \tag{20}$$

Note that while the parameters in the triggering functions incorporate the in-degree d_i, they are local network parameters that each agent can compute based on its own neighborhood information, without requiring global knowledge of the network topology. The constants κ and γ can be designed according to local information to satisfy the conditions in Formula (29). The proposed control method is detailed in Algorithm 1.

Algorithm 1 Edge-event-triggered sliding mode control for agent i

Initialization: Set initial state $x_i(0)$ (and $v_i(0)$ if $i\in I_S$), control parameters μ, ϖ, λ, and σ, triggering parameters κ, γ, ζ, and ι, and last trigger time $t_k^{ij}=0$ for all $j\in N_i$. Initialize $\hat{\xi}_{ij}(0)=x_i(0)-x_j(0)$ and $\hat{v}_{ij}(0)=v_i(0)$ (if $i\in I_S$).
Loop (at each control time t):
    For each neighbor $j\in N_i$:
        Calculate current relative position $\xi_{ij}(t)=x_i(t)-x_j(t)$.
        Calculate sampling error $\varepsilon_{ij}(t)=\hat{\xi}_{ij}(t)-\xi_{ij}(t)$.
        If agent $i\in I_S$:
            Calculate sampling error $\eta_{ij}(t)=\hat{v}_{ij}(t)-v_i(t)$.
            Check triggering condition:
            $h_{ij}(t)=\zeta(\kappa\varepsilon_{ij}^2(t)+\frac{\mu}{d_i}\eta_{ij}^2(t))-\left(\frac{\mu}{d_i}-\frac{1}{\kappa}\right)v_i^2(t)$.
        Else (agent $i\in I_{N-S}$):
            Calculate $\phi_i(t)=\sum_{k\in N_i}a_{ik}\xi_{ik}(t)$.
            Check triggering condition:
            $h_{ij}(t)=\iota\gamma\varepsilon_{ij}^2(t)-\left(\frac{2}{d_i}-\frac{1}{\gamma}\right)\phi_i^2(t)$.
        End If
        If $h_{ij}(t)\geqslant 0$:
            Update sampled value: $\hat{\xi}_{ij}(t)\leftarrow\xi_{ij}(t)$.
            If agent $i\in I_S$: $\hat{v}_{ij}(t)\leftarrow v_i(t)$. **End If**
            Record trigger time $t_{k+1}^{ij}=t$. Send updated information if necessary.
        End If
    End For
    Calculate integral sliding variable $s_i(t)$ using Eq. (15) or Eq. (16).
    Calculate equivalent control $u_i^{\mathrm{eq}}(t)$ using Eq. (10).
    Calculate sliding mode control $u_i^s(t)$ using Eq. (11).
    Calculate total control input $u_i(t)$ using Eq. (9).
    Apply $u_i(t)$ to agent i.
End Loop

Remark 1 The triggering conditions are different according to the types of agents. When $h_{ij}(t)\geqslant 0$, agent i will update $\hat{\xi}_{ij}$ (and $\hat{v}_{ij}$ if $i\in I_S$), and ε_{ij} (and η_{ij}) is reset to zero. If $i\in I_S$, the triggering condition h_{ij} involves the position error ε_{ij} and velocity error η_{ij} relative to its sampled state $v_i(t_k^{ij})$, enabling point-to-point communications triggered by edge-specific state deviations. If $i\in I_{N-S}$, due to the first-order dynamics, the triggering condition h_{ij} is independent of velocity sampling error and depends on the local consensus error $\phi_i(t)$.

Remark 2 The constant parameters ζ and ι in triggering functions can adjust the triggering frequency. With larger κ and ζ, the triggering frequency for agents with the second-order dynamics will increase. Similarly, larger γ and ι will lead to the increase of triggering time of the first-order agents. With the increase of triggering frequency, controller will update sampling instances more frequently.

Remark 3 By analyzing the edge-event-triggered consensus control function in Eq. (10), it is evident that the relative positions, rather than the absolute positions, of the heterogeneous MASs are utilized. This indicates that the consensus control protocol designed here does not require the absolute position of each agent. Every agent can possess its own independent local coordinates to accomplish the consensus control goal.

In Section IV, we will provide a rigorous theoretical analysis of the proposed control strategy, demonstrating its effectiveness in achieving consensus for heterogeneous MASs under unknown disturbances.

IV. Main Result

Stability analysis of the consensus of heterogeneous MASs under the SMC-based edge-event-triggered control protocol is provided in this section. First, we need to demonstrate that heterogeneous MASs can attain the sliding mode surfaces within a finite time period.

Theorem 1 With Assumption 2 and the control protocol in Eqs. (9)–(11), the states of heterogeneous MASs in Eqs. (1) and (2) will reach the sliding mode surface in Eqs. (15) and (16) in a finite time when $\sigma > \bar{\delta}$ and $0 < \lambda < 1$. The condition $\sigma > \bar{\delta}$ ensures that the sliding mode control component has sufficient control authority to overcome the worst-case disturbance. Besides, the setting time T_s has an upper bound given by

$$T_s \leqslant \frac{V_s(0)^{1-\lambda/2}}{\beta(1-\lambda/2)} \tag{21}$$

where $\beta = 2^{\lambda/2}\varpi$.

Proof Consider the following positive-definite Lyapunov function

$$V_s(t) = \frac{1}{2}\sum_{i=1}^{N} s_i^2 = \frac{1}{2}\sum_{i=1}^{N} s_i^{\mathrm{T}} s_i \tag{22}$$

Based on Eqs. (15) and (16), the derivative of $V_s(t)$ is described by

$$\begin{aligned}\dot{V}_s(t) = \sum_{i=1}^{N} s_i\dot{s}_i = \sum_{i=1}^{S} s_i(\dot{v}_i(t) + \sum_{j\in N_i} a_{ij}(\hat{\dot{\xi}}_{ij}(t)+ \\ \frac{1}{d_i}\mu\hat{v}_{ij}(t))) + \sum_{i=S+1}^{N} s_i(\dot{x}_i(t) + \sum_{j\in N_i} a_{ij}\hat{\dot{\xi}}_{ij}(t))\end{aligned} \tag{23}$$

By combining Eqs. (1), (2), (9), and (10), and substituting $\dot{v}_i = g_i u_i + f_i + \delta_i$ and $\dot{x}_i = g_i u_i + f_i + \delta_i$, along with $u_i = g_i^{-1}(u_i^{\mathrm{eq}} + u_i^s)$ and the definition of u_i^{eq} in Eq. (10), one has

$$\begin{aligned}\dot{V}_s(t) = \sum_{i=1}^{N} s_i(\delta_i(t) + u_i^s(t)) = \\ \sum_{i=1}^{N} s_i(\delta_i(t) - \varpi|s_i|^{\lambda}\mathrm{sgn}(s_i) - \sigma\mathrm{sgn}(s_i))\end{aligned} \tag{24}$$

With the condition $s_i\mathrm{sgn}(s_i) = |s_i|$, $|\delta_i(t)| \leqslant \bar{\delta}$, and Lemma 2, we acquire

$$\begin{aligned}\dot{V}_s(t) \leqslant \sum_{i=1}^{N}(|s_i||\delta_i(t)| - \varpi|s_i|^{\lambda+1} - \sigma|s_i|) \leqslant \sum_{i=1}^{N}|s_i|(\bar{\delta}-\sigma)- \\ \sum_{i=1}^{N}\varpi|s_i|^{\lambda+1} \leqslant -\varpi\sum_{i=1}^{N}|s_i|^{\lambda+1} \leqslant -\varpi\left(\sum_{i=1}^{N}|s_i|^2\right)^{(\lambda+1)/2} \leqslant \\ -\varpi(2V_s)^{(\lambda+1)/2} = -\beta' V_s^{(\lambda+1)/2}\dot{V}_s(t) = -\varpi(2V_s)^{\lambda/2} = \\ -\beta V_s^{\lambda/2}\end{aligned} \tag{25}$$

where $\beta = 2^{\lambda/2}\varpi$ is a positive constant. Formula (25) is rewritten as

$$\dot{V}_s + \beta V_s^{\lambda/2} \leqslant 0 \tag{26}$$

By Lemma 1, since $0 < \lambda < 1$ implies $0 < \lambda/2 < 1$, we can conclude that $V_s(t)$ meets the requirements of finite time stability. Thus, $s_i(t) \to 0$ in a finite time for all $i \in I_N$. The upper limit for the setting time can be calculated using Formula (5). ■

Remark 4 Theorem 1 suggests that $s_i = 0$ (and consequently $\dot{s}_i = 0$ can be achieved on the surface) will be attained within a finite time. When heterogeneous MASs reach the sliding mode designed in Eqs. (15) and (16), the disturbances are suppressed by the sliding mode control.

Remark 5 The SMC function $u_i^s(t)$ contains the sign function of s_i as well as the power term of s_i. The item $\varpi|s_i|^{\lambda}$ varies according to the value of s_i. As $|s_i|$ increases, the strength of the control signal $u_i^s(t)$ will also increase, accelerating the convergence speed of s_i. When s_i approaches zero, this item is almost zero, contributing less to chattering. Additionally, the item $\sigma\mathrm{sgn}(s_i)$ guarantees the convergence of s_i for small s_i values and counteracts the disturbance. Moreover, to alleviate the chattering phenomenon that can occur with the sign function, smooth approximations such as $\tanh(s_i/\varepsilon)$ can be employed, where ε is a small positive constant. This replacement creates a boundary layer around the sliding surface, within which the control varies continuously rather than discontinuously, significantly reducing chattering at the cost of a small tracking error.

Remark 6 As required by Assumption 2 and Theorem 1, the selection of the SMC gain σ requires knowledge of the disturbance upper bound $\bar{\delta}$ ($\sigma > \bar{\delta}$). In practical applications, estimating $\bar{\delta}$ can be challenging. Several approaches can be used. (1) Based on the physical modeling of the system, analyze possible disturbance sources (friction, wind, etc.) and conservatively estimate their maximum magnitudes. (2) Use adaptive techniques where σ is not fixed but adjusted online based on state information or estimation of disturbances [31]. (3) Implement a disturbance observer to estimate the actual disturbance $\delta_i(t)$ and adapt σ or use the estimate in the control law [6].

Once Eqs. (1) and (2) attain the sliding mode surface in Eqs. (15) and (16) within a finite duration, considering $\dot{s}_i(t) = 0$, the equivalent dynamics under the equivalent control part can be reformulated as

$$\begin{cases}\dot{x}_i(t) = v_i(t) \\ \dot{v}_i(t) = -\sum\limits_{j\in N_i} a_{ij}\left(\hat{\xi}_{ij}(t) + \dfrac{1}{d_i}\mu\hat{v}_{ij}(t)\right),\ i\in I_S\end{cases} \tag{27}$$

$$\dot{x}_i(t) = -\sum_{j\in N_i} a_{ij}\hat{\xi}_{ij}(t),\ i\in I_{N-S} \tag{28}$$

By analyzing Eqs. (27) and (28), it is obvious that the influence of matched disturbance $\delta_i(t)$ has been eliminated by the sliding mode control component $u_i^s(t)$. Then, we will prove the validity of the proposed edge-event-triggered mechanism for heterogeneous MASs consensus using the Lyapunov stability theory.

Theorem 2 Assume that Assumption 1 is satisfied. With the edge-event-triggered mechanism defined by Eq. (17) and the triggering function in Eqs. (18) and (19), the consensus problem of heterogeneous MASs with the dynamics in Eqs. (27) and (28) can be solved if the positive parameters satisfy the following conditions

$$\begin{aligned}&\frac{\mu}{d_i} - \frac{1}{\kappa} > 0,\ i\in I_S;\\ &\frac{2}{d_i} - \frac{1}{\gamma} > 0,\ i\in I_{N-S}\end{aligned} \tag{29}$$

where κ and γ are positive constants. Note that these parameters can be determined locally by each agent based on its own in-degree d_i, without requiring global network information.

Proof The Lyapunov function is chosen to incorporate both position consensus error and velocity terms, allowing us to analyze the overall system stability. The positive-definite Lyapunov function V is selected as

$$V(t) = \frac{1}{2}\sum_{i=1}^{N}\sum_{j=1}^{N} a_{ij}(x_i(t)-x_j(t))^2 + \sum_{i=1}^{S} v_i^2(t) \tag{30}$$

Differentiating $V(t)$ yields

$$\begin{aligned}\dot{V}(t) = &\sum_{i=1}^{N}\sum_{j=1}^{N} a_{ij}(x_i(t)-x_j(t))(\dot{x}_i(t)-\dot{x}_j(t)) + \\ &2\sum_{i=1}^{S} v_i(t)\dot{v}_i(t) = \\ &\sum_{i=1}^{S}\sum_{j=1}^{S} a_{ij}(x_i(t)-x_j(t))(v_i(t)-v_j(t)) + \\ &\sum_{i=1}^{S}\sum_{j=S+1}^{N} a_{ij}(x_i(t)-x_j(t))(v_i(t)-\dot{x}_j(t)) + \\ &\sum_{i=S+1}^{N}\sum_{j=1}^{S} a_{ij}(x_i(t)-x_j(t))(\dot{x}_i(t)-v_j(t)) + \\ &\sum_{i=S+1}^{N}\sum_{j=S+1}^{N} a_{ij}(x_i(t)-x_j(t))(\dot{x}_i(t)-\dot{x}_j(t)) + \\ &2\sum_{i=1}^{S} v_i(t)\dot{v}_i(t)\end{aligned} \tag{31}$$

Due to the symmetry of the adjacency matrix A, Eq. (31) can be transformed into

$$\begin{aligned}\dot{V}(t) = &2\sum_{i=1}^{S} v_i(t)(\dot{v}_i(t) + \sum_{j=1}^{N} a_{ij}(x_i(t)-x_j(t))) + \\ &2\sum_{i=S+1}^{N}\dot{x}_i(t)\sum_{j=1}^{N} a_{ij}(x_i(t)-x_j(t))\end{aligned} \tag{32}$$

In light of Eqs. (13), (14), and (27) and the Young's inequality ($xy \leqslant \kappa x^2/2 + y^2/(2\kappa)$ for any positive constant κ), we have

$$\begin{aligned}&2\sum_{i=1}^{S} v_i(t)(\dot{v}_i(t) + \sum_{j=1}^{N} a_{ij}(x_i(t)-x_j(t))) = \\ &2\sum_{i=1}^{S} v_i(t)\left(-\sum_{j=1}^{N} a_{ij}(\hat{\xi}_{ij}(t) + \frac{1}{d_i}\mu\hat{v}_{ij}(t)) + \right.\\ &\left.\sum_{j=1}^{N} a_{ij}\xi_{ij}(t)\right) = \\ &2\sum_{i=1}^{S}\sum_{j=1}^{N} a_{ij}v_i(t)(\xi_{ij}(t)-\hat{\xi}_{ij}(t)) - \\ &2\mu\sum_{i=1}^{S}\sum_{j=1}^{N}\frac{1}{d_i}v_i(t)(\eta_{ij}(t)+v_i(t)) \leqslant \\ &\sum_{i=1}^{S}\sum_{j=1}^{N} a_{ij}(\kappa\varepsilon_{ij}^2(t) + \frac{\mu}{d_i}\eta_{ij}^2(t) - \\ &\left(\frac{\mu}{d_i} - \frac{1}{\kappa}\right)v_i^2(t))\end{aligned} \tag{33}$$

For the second term in Eq. (32), combining Eqs. (13) and (28) and the Young's inequality, we can obtain

$$\begin{aligned}&2\sum_{i=S+1}^{N}\dot{x}_i(t)\sum_{j=1}^{N} a_{ij}(x_i(t)-x_j(t)) = \\ &-2\sum_{i=S+1}^{N}\left(\sum_{j=1}^{N} a_{ij}\hat{\xi}_{ij}(t)\right)\left(\sum_{k=1}^{N} a_{ik}\xi_{ik}(t)\right) = \\ &-2\sum_{i=S+1}^{N}\left(\sum_{j=1}^{N} a_{ij}(\xi_{ij}(t)+\varepsilon_{ij}(t))\right)\phi_i(t) = \\ &-2\sum_{i=S+1}^{N}\phi_i^2(t) - 2\sum_{i=S+1}^{N}\sum_{j=1}^{N} a_{ij}\varepsilon_{ij}(t)\phi_i(t) \leqslant \\ &-2\sum_{i=S+1}^{N}\phi_i^2(t) + \sum_{i=S+1}^{N}\sum_{j=1}^{N} a_{ij}\left(\gamma\varepsilon_{ij}^2(t) + \frac{1}{\gamma d_i^2}(d_i\phi_i(t))^2\right) \leqslant \\ &-2\sum_{i=S+1}^{N}\sum_{j=1}^{N} a_{ij}\frac{1}{d_i}\phi_i^2(t) + \sum_{i=S+1}^{N}\sum_{j=1}^{N} a_{ij}\left(\gamma\varepsilon_{ij}^2(t) + \frac{1}{\gamma}\phi_i^2(t)\right) = \\ &\sum_{i=S+1}^{N}\sum_{j=1}^{N} a_{ij}\left[\gamma\varepsilon_{ij}^2(t) - \left(\frac{2}{d_i} - \frac{1}{\gamma}\right)\phi_i^2(t)\right]\end{aligned} \tag{34}$$

This results in the inequality

$$\begin{aligned}&2\sum_{i=S+1}^{N}\dot{x}_i(t)\sum_{j=1}^{N}a_{ij}(x_i(t)-x_j(t))\leqslant\\&\sum_{i=S+1}^{N}\sum_{j=1}^{N}a_{ij}\left[\gamma\varepsilon_{ij}^2(t)-\left(\frac{2}{d_i}-\frac{1}{\gamma}\right)\phi_i^2(t)\right]\end{aligned}\tag{35}$$

where $\phi_i(t)=\sum_{j=1}^{N}a_{ij}\xi_{ij}(t)$ and γ is a positive constant.

Substituting Formulas (33) and (35) into Eq. (32) yields

$$\begin{aligned}\dot{V}(t)\leqslant&\sum_{i=1}^{S}\sum_{j=1}^{N}a_{ij}\left[\kappa\varepsilon_{ij}^2(t)+\frac{\mu}{d_i}\eta_{ij}^2(t)-\left(\frac{\mu}{d_i}-\frac{1}{\kappa}\right)v_i^2(t)\right]+\\&\sum_{i=S+1}^{N}\sum_{j=1}^{N}a_{ij}\left[\gamma\varepsilon_{ij}^2(t)-\left(\frac{2}{d_i}-\frac{1}{\gamma}\right)\phi_i^2(t)\right]\end{aligned}\tag{36}$$

In light of the edge-event-triggered mechanisms in Eqs. (17)–(19) (which ensuring the terms involving v_i^2 and ϕ_i^2 are upper bounded by the error terms at trigger instants), we can further obtain

$$\begin{aligned}\dot{V}(t)\leqslant&-\sum_{i=1}^{S}\sum_{j=1}^{N}a_{ij}(\zeta-1)\left(\kappa\varepsilon_{ij}^2(t)+\frac{\mu}{d_i}\eta_{ij}^2(t)\right)-\\&\sum_{i=S+1}^{N}\sum_{j=1}^{N}a_{ij}(\iota-1)\gamma\varepsilon_{ij}^2(t)\leqslant 0\end{aligned}\tag{37}$$

Utilizing the Lyapunov stability theory and Formula (37), $\dot{V}(t)\leqslant 0$. By the LaSalle's invariance principle, the trajectories converge to the largest invariant set where $\dot{V}(t)=0$, which implies $\varepsilon_{ij}(t)=0$ and $\eta_{ij}(t)=0$ (for $i\in I_S$). This means $\hat{\xi}_{ij}(t)=\xi_{ij}(t)$ and $\hat{v}_{ij}(t)=v_i(t)$. Substituting these into the equivalent dynamics in Eqs. (27) and (28) and analyzing the resulting closed-loop system shows that $\lim_{t\to\infty}|x_i(t)-x_j(t)|=0$ for all i and j if the graph is connected (Assumption 1), and $\lim_{t\to\infty}|v_i(t)|=0$ for $i\in I_S$. The control objective in Eq. (3) is satisfied. ■

Theorems 1 and 2 demonstrate that heterogeneous MASs in Eqs. (1) and (2) attain consensus with the control strategy in Eqs. (9)–(11). Then, we need to exclude the Zeno behavior.

Theorem 3 The utilization of edge-event-triggered mechanisms and triggering functions eliminates the Zeno behavior (as defined in Definition 2).

Proof We analyze the Zeno behavior of edge (i,j) by showing that the inter-event time $t_{k+1}^{ij}-t_k^{ij}$ is lower bounded by a positive constant. This is done in four cases according to the different types of the agents making up the edges.

Case 1: When $i\in I_S$ and $j\in I_S$, we let $e_{ij}(t)=\sqrt{\kappa}|\varepsilon_{ij}(t)|+\sqrt{\frac{\mu}{d_i}}|\eta_{ij}(t)|$. For $t\in[t_k^{ij},t_{k+1}^{ij})$, the right-hand derivative of $e_{ij}(t)$ satisfies

$$\begin{aligned}D^+(e_{ij}(t))\leqslant&\sqrt{\kappa}|\dot{\varepsilon}_{ij}(t)|+\sqrt{\frac{\mu}{d_i}}|\dot{\eta}_{ij}(t)|=\\&\sqrt{\kappa}|\dot{x}_i(t)-\dot{x}_j(t)|+\sqrt{\frac{\mu}{d_i}}|\dot{v}_i(t)|=\\&\sqrt{\kappa}|v_i(t)-v_j(t)|+\sqrt{\frac{\mu}{d_i}}|\dot{v}_i(t)|\leqslant\\&\sqrt{\kappa}(|v_i(t)|+|v_j(t)|)+\\&\sqrt{\frac{\mu}{d_i}}|\sum_{p\in N_i}a_{ip}(\hat{\xi}_{ip}(t)+\\&\frac{\mu}{d_i}\hat{v}_{ip}(t))+u_i^s(t)+\delta_i(t)|\end{aligned}\tag{38}$$

According to Theorem 2, the states $x_i(t)$ and $v_i(t)$ are bounded. Since $\hat{\xi}_{ij}(t)$ and $\hat{v}_{ij}(t)$ are sampled values from bounded states, they are also bounded between events. The sliding variable $s_i(t)$ is bounded (converging to zero), thus $u_i^s(t)$ defined in Eq. (11) is bounded. The disturbance $\delta_i(t)$ is bounded by Assumption 2. Therefore, $D^+(e_{ij}(t))$ is upper bounded by a positive constant $\tau_{ij}^{(1)}$.

$$D^+(e_{ij}(t))\leqslant\tau_{ij}^{(1)}\tag{39}$$

Integrating Formula (39) from t_k^{ij} to t ($t<t_{k+1}^{ij}$) gives $e_{ij}(t)-e_{ij}(t_k^{ij})\leqslant\tau_{ij}^{(1)}(t-t_k^{ij})$. Since $e_{ij}(t_k^{ij})=0$ (errors are reset at trigger time), we have $e_{ij}(t)\leqslant\tau_{ij}^{(1)}(t-t_k^{ij})$. At the next trigger time $t=t_{k+1}^{ij}$, the trigger condition in Eq. (18) must hold, implying $h_{ij}(t_{k+1}^{ij})\geqslant 0$. This gives

$$\kappa\varepsilon_{ij}^2(t_{k+1}^{ij})+\frac{\mu}{d_i}\eta_{ij}^2(t_{k+1}^{ij})\geqslant\frac{1}{\zeta}\left(\frac{\mu}{d_i}-\frac{1}{\kappa}\right)v_i^2(t_{k+1}^{ij})\tag{40}$$

Using the inequality $(a+b)^2\geqslant a^2+b^2$ for $a,b\geqslant 0$, we have

$$\begin{aligned}e_{ij}^2(t_{k+1}^{ij})\geqslant&\kappa\varepsilon_{ij}^2(t_{k+1}^{ij})+\frac{\mu}{d_i}\eta_{ij}^2(t_{k+1}^{ij})\geqslant\\&\frac{1}{\zeta}\left(\frac{\mu}{d_i}-\frac{1}{\kappa}\right)v_i^2(t_{k+1}^{ij})\triangleq C_{ij}v_i^2(t_{k+1}^{ij})>0\end{aligned}\tag{41}$$

where $C_{ij}=\frac{1}{\zeta}\left(\frac{\mu}{d_i}-\frac{1}{\kappa}\right)>0$ by condition in Formula (29). Combining $e_{ij}(t_{k+1}^{ij})\leqslant\tau_{ij}^{(1)}(t_{k+1}^{ij}-t_k^{ij})$ and $e_{ij}(t_{k+1}^{ij})\geqslant\sqrt{C_{ij}}$ $|v_i(t_{k+1}^{ij})|$, we get

$$t_{k+1}^{ij}-t_k^{ij}\geqslant\frac{e_{ij}(t_{k+1}^{ij})}{\tau_{ij}^{(1)}}\geqslant\frac{\sqrt{C_{ij}}|v_i(t_{k+1}^{ij})|}{\tau_{ij}^{(1)}}\tag{42}$$

If $v_i(t)$ does not converge to zero instantaneously, the inter-event time is lower bounded by a positive value. A more rigorous argument shows that $e_{ij}(t)$ must grow to a positive value to trigger the event, guaranteeing $t_{k+1}^{ij}-t_k^{ij}>0$. The triggering interval has a positive lower bound.

Case 2: When $i\in I_S$ and $j\in I_{N-S}$, similar analysis applies. $D^+(e_{ij}(t))$ involves $\dot{x}_j(t)$ instead of $v_j(t)$. Since $\dot{x}_j(t)$ is bounded (depending on control u_j and disturbance δ_j), $D^+(e_{ij}(t))$ is bounded by some $\tau_{ij}^{(2)}>0$. The trigger condition is in Eq. (18), leading to the same form of lower bound as Formula (42).

Case 3: When $i\in I_{N-S}$ and $j\in I_S$, we define

$e_{ij}(t) = \sqrt{\gamma}|\varepsilon_{ij}(t)|$. Then $D^+(e_{ij}(t)) \leqslant \sqrt{\gamma}|\dot{\varepsilon}_{ij}(t)| = \sqrt{\gamma}|\dot{x}_i(t) - \dot{x}_j(t)| = \sqrt{\gamma}|\dot{x}_i(t) - v_j(t)|$. Since $\dot{x}_i$ and v_j are bounded, $D^+(e_{ij}(t))$ is bounded by some $\tau_{ij}^{(3)} > 0$. The trigger condition is in Eq. (19), $h_{ij}(t_{k+1}^{ij}) \geqslant 0$, which means

$$\gamma\varepsilon_{ij}^2(t_{k+1}^{ij}) \geqslant \frac{1}{\iota}\left(\frac{2}{d_i} - \frac{1}{\gamma}\right)\phi_i^2(t_{k+1}^{ij}) \tag{43}$$

Therefore, $e_{ij}^2(t_{k+1}^{ij}) \geqslant \frac{1}{\iota}\left(\frac{2}{d_i} - \frac{1}{\gamma}\right)\phi_i^2(t_{k+1}^{ij}) \triangleq C'_{ij}\phi_i^2(t_{k+1}^{ij}) > 0$.

Combining with $e_{ij}(t_{k+1}^{ij}) \leqslant \tau_{ij}^{(3)}(t_{k+1}^{ij} - t_k^{ij})$, we get

$$t_{k+1}^{ij} - t_k^{ij} \geqslant \frac{e_{ij}(t_{k+1}^{ij})}{\tau_{ij}^{(3)}} \geqslant \frac{\sqrt{C'_{ij}}|\phi_i(t_{k+1}^{ij})|}{\tau_{ij}^{(3)}} > 0 \tag{44}$$

Case 4: When $i \in I_{N-S}$ and $j \in I_{N-S}$, similar analysis as case 3 is applied. $D^+(e_{ij}(t)) \leqslant \sqrt{\gamma}|\dot{x}_i(t) - \dot{x}_j(t)|$, which is bounded by some $\tau_{ij}^{(4)} > 0$. The trigger condition and lower bound have the same form as Formula (44).

By summarizing the proof in four cases, a positive lower bound exists for the triggering interval of all types of edges. Therefore, there is no Zeno behavior in the edge-event-triggered mechanism protocol designed in this paper. ■

V. Simulation Result and Analysis

In this section, we present comprehensive numerical simulations to validate the proposed edge-event-triggered sliding mode consensus control strategy for heterogeneous MASs. The simulations demonstrate the performance, robustness, and communication efficiency of our approach for a system consisting of both the first-order and second-order agents under unknown disturbances.

A. Simulation Setup

The heterogeneous MASs considered in the simulation comprise ten agents ($N = 10$), with five second-order agents ($S = 5$, agents 1–5) and five first-order agents ($N - S = 5$, agents 6–10). The communication topology is shown in Fig. 1. The initial position of the ten agents is set as $x(0) = [20, 15, 10, 5, 0, -5, -10, -15, -20, -25]$, and the initial velocity of the second-order agents is $v(0) = [4, -3, 2, -1, 0]$. For the baseline simulation, the unknown disturbance is modeled as $\delta_i(t) = 0.12\sin(0.3t + i/3)$ for $i \in I_S = \{1, 2, ..., 5\}$, and $\delta_i(t) = 0.1\sin(0.2t + i/5)$ for $i \in I_{N-S} = \{6, 7, ..., 10\}$. This implies a disturbance bound $\bar{\delta} = 0.12$.

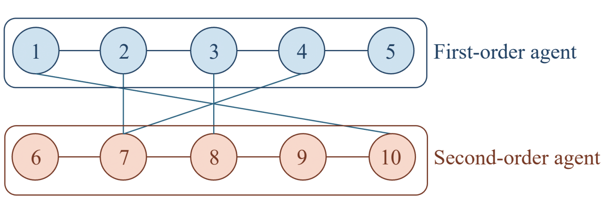

Figure 1 Communication topology of the heterogeneous MASs with 10 agents.

The controller parameters are set as $\mu = 3$, $\varpi = 2$, $\lambda = 0.8$, and $\sigma = 0.8$. The edge-event-triggered mechanism parameters are chosen as $\kappa = 16$, $\gamma = 15$, $\zeta = 1$, and $\iota = 1$. These values satisfy the conditions in Formula (29) (e.g., for $d_i = 3$, $\mu/d_i - 1/\kappa = 3/3 - 1/16 > 0$, and $2/d_i - 1/\gamma = 2/3 - 1/15 > 0$) and $\sigma > \bar{\delta}$, ensuring theoretical guarantees. The simulation uses a step size $r = 0.005$ s.

B. Consensus Achievement

The position trajectories of all agents under the proposed EET-SMC strategy (with sinusoidal disturbances) are displayed in Fig. 2. Despite the heterogeneous nature of the system and the presence of unknown disturbances, all agents converge to a common position value, confirming the effectiveness of our approach in achieving consensus. The system exhibits a smooth convergence behavior.

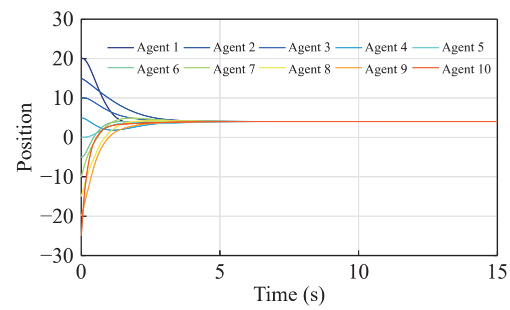

Figure 2 Position trajectory of agent (EET-SMC with sinusoidal disturbance).

Figure 3 presents the velocity trajectories of the second-order agents. In accordance with the control objectives in Eq. (3), these velocities asymptotically converge to zero, aligning with the static consensus condition. This demonstrates the capability of our control protocol to coordinate agents with different order dynamics. The consensus error evolution is illustrated in Fig. 4, where the consensus error is defined as $\sqrt{\sum_{i=1}^{N}\sum_{j=i+1}^{N}(x_i(t) - x_j(t))^2}$. The error decreases rapidly and approaches zero, providing quantitative confirmation of consensus achievement and validating the theoretical analysis.

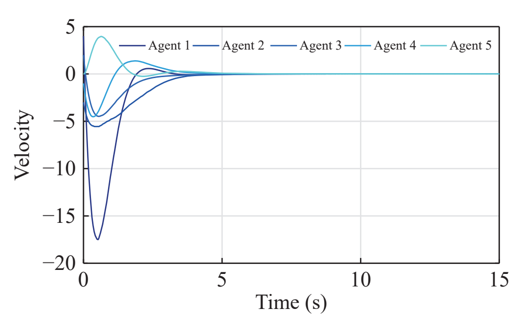

Figure 3 Velocity trajectory of the second-order agent (EET-SMC with sinusoidal disturbance).

C. Edge-Event-Triggered Mechanism Analysis

The triggering instants for all communication edges under EET-SMC are depicted in Fig. 5. Each point indicates an event where an agent samples the state of its neighbor due to the edge condition in Eq. (18) or Eq. (19) being met. The distribution of events across time confirms the absence of Zeno behavior, with finite positive intervals between consecutive triggers for each edge, as proven in Theorem 3. Notably, the triggering frequency adapts to the system state, meaning that more frequent triggering occurs during the initial transient phase and gradually reduces as the system approaches consensus.

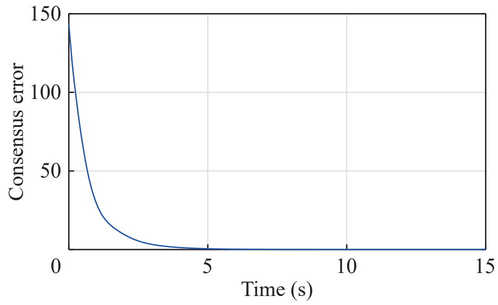

Figure 4 Consensus error evolution (EET-SMC with sinusoidal disturbance).

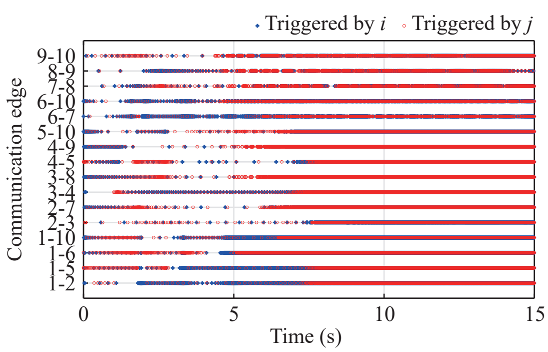

Figure 5 Triggering instant for communication edge (EET-SMC).

D. Robustness to Stochastic Disturbance

To evaluate the performance of controller under stochastic disturbances, we simulate the system using the proposed EET-SMC but replace the sinusoidal disturbance $\delta_i(t)$ with band-limited white noise. The noise is generated by passing zero-mean Gaussian white noise through a first-order low-pass filter and scaling it such that its magnitude is effectively bounded by $\bar{\delta}_{\text{noise}} = 0.15$. The controller parameter $\sigma = 0.8$ remains larger than this bound. Figure 6 shows the position trajectories and Fig. 7 shows the consensus error evolution under these noise conditions.

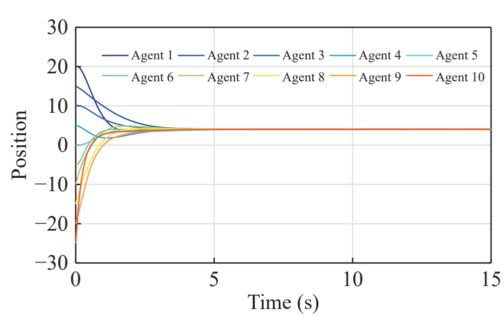

Figure 6 Position trajectory under band-limited white noise disturbance (EET-SMC).

As observed, the positions of agents still converge towards a common value, and the consensus error converges to a small

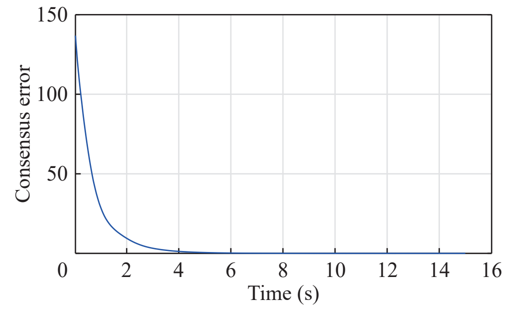

Figure 7 Consensus error evolution under band-limited white noise disturbance (EET-SMC).

residual set around zero. This demonstrates that the proposed EET-SMC maintains practical consensus and robustness even when disturbances are stochastic, providing that their effective bound is handled by the SMC gain σ.

E. Control Signal Evolution

Figure 8 displays the control input signals $u_i(t)$ for representative agents (agent 1, second-order and agent 6, first-order) under the EET-SMC with sinusoidal disturbances.

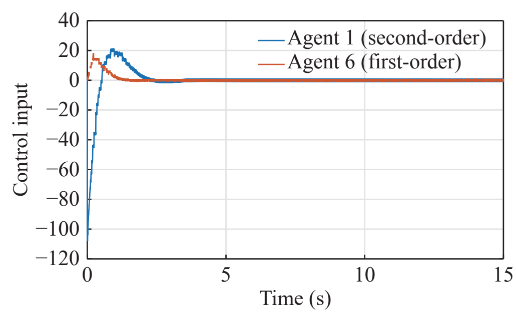

Figure 8 Control input signals $u_1(t)$ and $u_6(t)$ (EET-SMC with sinusoidal disturbance).

Figure 8 shows the control activity required to achieve consensus and reject disturbances. The initial phase exhibits a larger control effort. The effect of the sliding mode component u_i^s is visible. Some high-frequency switching (chattering) associated with the sign function in u_i^s may be observed, which can be mitigated in practice using approximations as discussed in Remark 6. The event-triggered nature means that the control signal changes based on sampled information updated only at trigger instants.

F. Comparative Analysis

To further demonstrate the advantages of the proposed EET-SMC strategy, we conduct a comparative simulation against two baseline methods under the same sinusoidal disturbance setup.

(1) Node-based event-triggered SMC (NBET-SMC): Use the same SMC law but employ a standard node-based triggering mechanism where agent i triggers communication to all neighbors $j \in N_i$ if a node-level error exceeds a threshold.

(2) Continuous-time SMC (CT-SMC): Use the same SMC law but assume continuous communication, i.e., $\hat{\xi}_{ij}(t) = \xi_{ij}(t)$ and $\hat{v}_{ij}(t) = v_i(t)$ at all times.

The results in Fig. 9 indicate that while all methods achieve consensus, the proposed EET-SMC significantly reduces communication compared to NBET-SMC while achieving comparable convergence speed and control effort to CT-SMC, demonstrating its effectiveness and efficiency. The NBET-SMC requires much more communication than EET-SMC for a similar performance.

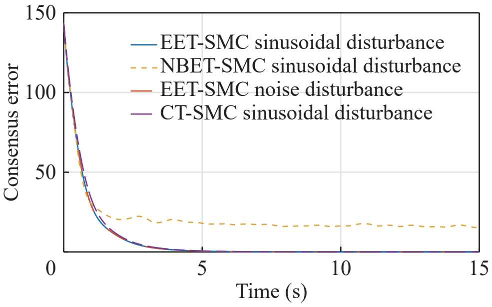

Figure 9 Consensus error comparison.

G. Communication Efficiency

Detailed communication statistics from this specific comparison reveal substantial efficiency improvements with our edge-event-triggered approach. The total number of communications (sum of triggers over all edges) for the edge-based mechanism is 5902, compared to 15304 for the node-based approach (sum of triggers per node times node degree). This represents a communication reduction of 9402 messages or 61.31%. The average number of triggering events per edge is 368.88 for EET-SMC, while the estimated average communication event triggered per node is 1530.40 for NBET-SMC. These results quantitatively demonstrate the superior communication efficiency of our approach. Table 1 summarizes the key communication metrics for this comparison.

Table 1 Communication efficiency comparison.

Method	Total communication	Average communication per edge	Average communication per node	Communication reduction
EET-SMC	5902	368.88	590.00	9402 (61.31%)
NBET-SMC	15,304	—	1530.40	

H. Discussion

The simulation results validate the theoretical findings and demonstrate several key advantages of the proposed EET-SMC strategy.

(1) Consensus achievement: The approach successfully achieves consensus for heterogeneous MASs with mixed first-order and second-order dynamics, even in the presence of unknown disturbances.

(2) Disturbance rejection: The sliding mode control component effectively suppresses the influence of unknown matched disturbances, as evidenced by the stable consensus.

(3) Communication efficiency: The edge-event-triggered mechanism significantly reduces communication overhead compared to a comparable node-based approach, achieving 61.31% reduction in communication events in the simulation.

(4) No Zeno behavior: The positive intervals between consecutive trigger events confirm the absence of Zeno behavior, validating Theorem 3 and ensuring practical implementation.

These results highlight the effectiveness of our integrated approach, which combines the robustness of sliding mode control with the efficiency of edge-event-triggered mechanisms to achieve consensus in heterogeneous MASs while minimizing communication requirements.

VI. Conclusion

In this paper, the consensus control problem of heterogeneous MASs with unknown disturbances is addressed. A novel edge-event-triggered sliding mode control law is designed to achieve consensus of heterogeneous MASs consisting of the first-order and second-order agents. Using SMC, unknown disturbances are effectively addressed, and the heterogeneous MASs states are driven to the designed sliding surfaces in a finite time. The proposed edge-event-triggered approach offers significant advantages over traditional node-based methods by enabling selective communication between specific agent pairs, resulting in substantial reduction (61.31%) in communication frequency as demonstrated in our simulations. Comparisons with continuous-time control and alternative event-triggering schemes further highlight the efficiency and performance benefits of the proposed EET-SMC. Considering higher-order heterogeneous MASs with more complex dynamics is an important direction for future research.

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