Event-Based Adaptive Dynamic Programming for Switched Nonlinear Multi-Agent System Based on ISMC

Hao Jiang, Qinglai Wei, and Yongting Chen

***Abstract*—This article presents a robust optimal consensus control strategy for switched nonlinear multi-agent systems (MASs). The robust control problem with unmatched uncertainties is first reformulated as an optimal control framework through the introduction of augmented control inputs and a tailored cost function. To construct a distributed integral sliding-mode control (ISMC), a novel measurement error term is designed to enforce finite-time convergence of the consensus error to the origin. An event-based mechanism is incorporated to minimize controller execution, thereby optimizing computational and communication resource utilization. A concurrent learning approach is proposed to update neural network (NN) weights, where the value function is approximated by a critic NN. This method circumvents the constraints of initial admissible control and persistent excitation (PE) conditions inherent in traditional adaptive dynamic programming (ADP) designs. Theoretical analysis demonstrates that the developed event-triggered ADP controller guarantees system robustness for nonlinear MASs and ensures the uniform ultimate boundedness (UUB) of critic NN weight estimation errors. Finally, simulation results are presented to validate the effectiveness of the proposed control methodology.**

***Index Terms*—Adaptive dynamic programming (ADP), robust control, integral sliding-mode control (ISMC), event-based control, nonlinear multi-agent system (MAS)**

I. Introduction

In recent years, consensus control in multi-agent systems (MASs) has attracted increasing attention owing to its wide-ranging applications across various domains. Notable examples include formation control, sensor networks, flocking/swarming behaviors, and battery management systems, where coordinating multiple agents to reach a consensus is essential for achieving specific operational objectives. As research on MASs deepens, consensus control has emerged as a core challenge in distributed control theory [1, 2], with its influence spanning multiple disciplines. This has sparked intensive discussions and scholarly interest, reflecting the growing importance of consensus-based approaches in both theoretical research and practical engineering. Switched system [3, 4], a distinct category of hybrid systems, is composed of a finite set of subsystems orchestrated by a switching signal. These systems exhibit unique characteristics that distinguish them from conventional linear systems. For instance, Ref. [5] proposed an adaptive neural predefined-time control framework for switched under-actuated systems. Reference [6] addressed the comprehensive issues of fault-tolerant control and fault reconstruction for switched fuzzy systems. Particularly, because of the wide application in practice, the consensus problem of MASs containing switched features has attracted the attention of researchers [7, 8]. Reference [9] investigated the consensus control problem for a class of switched MASs under DoS attacks. The distributed control problem of an adaptive neural identifier for switched MASs was assessed in Ref. [10].

It is worth emphasizing that the aforementioned results solely ensure the stability of the MASs. Nevertheless, key control indices in practical applications, such as energy consumption and production costs, have not been considered. Therefore, optimal consensus control has attracted extensive attention in recent research [11]. Its core goal is to utilize the local information of each agent and its adjacent agents to design a distributed control protocol, thereby enabling all agents to achieve consensus while optimizing the overall control performance. In traditional nonlinear optimal control problems, solving the Hamilton-Jacobi-Bellman (HJB) equation presents a significant challenge. Adaptive dynamic programming (ADP) has been extensively investigated to address the "curse of dimensionality" in dynamic programming for coupled HJB equations [12, 13]. For instance, Ref. [14] developed the consensus control approach for MASs based on the ADP. In Ref. [15], the optimal consensus control problem of discrete-time MASs was investigated.

In general, traditional time-triggered approach involves transmitting massive amounts of data. Moreover, a large volume of transmitted data often impose a heavy burden on computing. To address the issue of unnecessary communication resource consumption, event-triggered control (ETC) has become an important research hotspot in the field

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of control. For example, the work in Ref. [16] delved into the security challenges of distributed consensus estimation for nonlinear systems under deception attacks. The adaptive output-feedback tracking control problem based on dynamic event-triggered for the nonlinear MASs with time-varying input delay was concerned in Ref. [17].

Sliding-mode control (SMC) [18, 19], as an effective control strategy for nonlinear systems, has been extensively applied to uncertain systems. Nevertheless, during the reaching phase, it lacks robustness against parameter variations and external matched disturbances. Integral sliding-mode control (ISMC) [20] effectively eliminates the arrival phase, thereby ensuring complete robustness throughout the control process of the entire closed-loop system. In Ref. [21], the stability of nonlinear MASs with multiple delays was dealt with. The finite-time resilient ISMC problem of stochastic systems under DoS attacks was studied in Ref. [22]. Notably, despite the progress in related fields, the design of event-based robust optimal consensus controllers for switched nonlinear MASs under single-critic ADP architectures remains under-explored.

Inspired by the above observations, we aim to explore a novel event-triggered robust optimal consensus control strategy for switched nonlinear MASs. The proposed approach is designed to enhance control response efficiency, minimize control costs, and optimize communication resource utilization. The key contributions of this work can be summarized in four fundamental aspects as follows:

(1) Different from the existing SMC in Refs. [18, 19], we propose a finite-time ISMC method with a single-critic ADP structure for switched nonlinear MASs. This approach not only eliminates the impact of matched disturbances but also guarantees the finite-time reachability of the sliding-mode surface (SMS).

(2) In contrast to the optimal control framework presented in Refs. [13, 23, 24], we introduce an improved cost function and augmented control input to compensate for unmatched uncertainties in sliding-mode dynamics.

(3) In contrast to conventional ETC, we employ a state-based ETC that acquires event-sampled state variables for control design. This approach reduces communication overhead from sensors to the controller by leveraging only event-triggered state updates. Additionally, the control input is updated using these event-sampled states, minimizing communication resources required between the controller and actuators.

(4) Compared with the actor-critic structure in Refs. [25, 26], a critic-only neural network (NN) framework is proposed to approach the optimal control strategy and solve the CHJB equation. Notably, by incorporating supplementary stability terms into the control strategy, the necessity for an initial admissible control prerequisite is eliminated. Additionally, the concurrent learning technique adopted herein effectively replaces the traditional persistent excitation (PE) condition.

II. Problem Formulation

In this section, we present a concise overview of graph theory and problem description.

A. Graph Theory

The directed graph can be represented as $G=(\hbar,\ell)$, where $\ell\subseteq\hbar\times\hbar$ denotes the edge set and $\hbar=\{\delta_1,\delta_2,...,\delta_N\}$ is the node set. Therefore, the neighbor set of agent i is defined as $M_i=\{\delta_j|(\delta_j,\delta_i)\in\ell\}$. If the weights of edges are considered, the graph is called a weighted graph. The adjacency matrix $A=[\alpha_{ij}]\in\mathbb{R}^{N\times N}$ serves as a common tool for describing the graph topology: $\alpha_{ij}>0$ is defined when the edge $e_{ji}=(\delta_j,\delta_i)\in\ell$, otherwise $\alpha_{ij}=0$. The in-degree matrix $D=\mathrm{diag}(\beta_1,\beta_2,...,\beta_N)\in\mathbb{R}^{N\times N}$ characterizes the in-degrees of nodes in the directed graph, where $d_i=\sum_{j=1}^{N}\alpha_{ij}$ denotes the in-degree of node δ_i. The Laplacian matrix is then defined as $L=D-A$, and its row sums are all zero.

B. Problem Description

Consider a class of nonlinear MASs consisting of $N+1$ agents with N $(N\geqslant 2)$ followers and one leader. The dynamic model of each follower is described as

$$\dot{x}_i=g^{\zeta(t)}(x_i)+f_i^{\zeta(t)}(x_i)u_i(t)+\rho_i(x_i)\varphi_i(t) \tag{1}$$

where $x_i\in\mathbb{R}^n$ and $u_i\in\mathbb{R}^m$ represent the state and control input of agent i, respectively. $g^{\zeta(t)}(x_i)\in\mathbb{R}^n$, $f_i^{\zeta(t)}(x_i)\in\mathbb{R}^{n\times m}$, and $\lambda_i(x_i)\in\mathbb{R}^{n\times 0}$ are known dynamics functions with $g^{\zeta(t)}(0)=0$. $\zeta(t):[0,+\infty)\to\omega=\{1,2,\ldots,\mathcal{M}\}$ is the switching signal, $\forall k\in\omega$, $\zeta(t)=k$ signifies the k-th active subsystem.

The leader x_0 dynamic is defined as

$$\dot{x}_0=f(x_0) \tag{2}$$

Assumption 1 The known control matrix is constrained by the condition that $\|f_i^k(x_i)\|\leqslant\Gamma_{fi}$. Then, $f_i^{k+}(x_i)\overset{\Delta}{=}(f_i^{k\mathrm{T}}(x_i)f_i^{k\mathrm{T}}(x_i)^{-1})^{-1}f_i^{k\mathrm{T}}(x_i)$ and is assumed to be bounded by $\|f_i^{k+}(x_i)\|\leqslant\Gamma_{fi}^{k+}$, where $f_i^{k+}(x_i)$ is the pseudo inverse of $f_i^k(x_i)$. $\rho_i(x_i)$ is bounded as $\|\rho_i(x_i)\|\leqslant\Gamma_{\rho_i}$. Furthermore, Γ_{fi}, Γ_{fi}^{+}, and Γ_{ρ_i} are known positive constants.

Assumption 2 The external impact $\varphi_i(t)$ is subject to a known bound $\varphi_{i\varphi}$, ensuring that $|\varphi_i(t)|\leqslant\varphi_{i\varphi}$, where $\varphi_{i\varphi}>0$. Furthermore, the expression $\rho_i(x_i)\varphi_i(t)$ can be broken down into two components: unmatched disturbances and matched disturbances.

$$\begin{aligned}\rho_i(x_i)\varphi_i(t)=&f_i^k(x_i)f_i^{k+}(x_i)\rho_i(x_i)\varphi_i(t)+\\&(\mathcal{I}-f_i^k(x_i)f_i^{k+}(x_i)\rho_i(x_i))\varphi_i(t)\end{aligned} \tag{3}$$

where $\mathcal{I}$ represents the appropriately adjusted identity matrix, $f_i^{k+}(x_i)\rho_i(x_i)\varphi_i(t)=\eta_i$, and $(\mathcal{I}-f_i^k(x_i)f_i^{k+}(x_i)\rho_i(x_i))=\mu_i(x_i)$. In addition, the dynamics $\mu_i(x_i)$ and matched disturbances η_i are also bounded, i.e., $\|\mu_i(x_i)\|\leqslant\Gamma_{\mu i}$ and $\|\eta_i\|\leqslant\Gamma_{\eta_i}$ with $\Gamma_{\mu i}>0$ and $\Gamma_{\eta_i}>0$.

To handle the mismatched disturbances, we consider the following candidate auxiliary nonlinear MAS

$$\dot{x}_i=g^k(x_i)+f_i^k(x_i)u_i(t)+f_i^k(x_i)\eta_i(t)+\mu_i(x_i)\varphi_i(t) \tag{4}$$

where $u_i(x_i)$ and $\varphi_i(t)$ represent the control variables.

The local neighborhood consensus error z_i of the i-th agent is characterized within the network topology of MAS.

$$z_i=\sum_{j\in N_i}a_{ij}(x_i-x_j)+b_i(x_i-x_0) \tag{5}$$

where $z_i=[z_{i1},z_{i2},\ldots,z_{in}]^{\mathrm{T}}\in\mathbb{R}^n$ and $b_i\geqslant 0$.

Thus, the core objective of this paper is to derive the local neighborhood error dynamics in Eq. (4) for addressing the optimal consensus control problem. Additionally, we establish a modified cost function as

$$J_i(z_i)=\int_t^{\infty}(\mathcal{Y}_1(x(\tau))+\mathcal{Y}_2(x(\tau))+\mathcal{Y}_3(x(\tau)))\mathrm{d}\tau \tag{6}$$

where

$$\begin{aligned}&\mathcal{Y}_1(x(\tau))=z_i^{\mathrm{T}}Q_{ii}z_i+\sigma_2\varphi_i^{\mathrm{T}}P_{ii}\varphi_i,\\&\mathcal{Y}_2(x(\tau))=\sigma_1\Gamma_{\eta i}^{\mathrm{T}}P_{ii}\Gamma_{\eta i}+\sigma_2\varphi_{i\lambda}^{\mathrm{T}}P_{ii}\varphi_{i\varphi},\\&\mathcal{Y}_3(x(\tau))=u_i^{\mathrm{T}}R_{ii}u_i+\sum_{j\in N_i}u_j^{\mathrm{T}}R_{ij}u_j\end{aligned} \tag{7}$$

where Q_{ii}, P_{ii}, and R_{ii} are symmetric positive definite matrices with appropriate dimensions, and $\sigma_1\geqslant 0$ and $\sigma_2\geqslant 0$ are parameters to be designed.

Assuming that $J_i(z_i)$ is continuous and differentiable, we deduce that

$$\mathcal{Y}_1(x)+\mathcal{Y}_2(x)+\mathcal{Y}_3(x)+J_{z_i}^{\mathrm{T}}\dot{z}_i=0 \tag{8}$$

Hence, the Hamiltonian function may be construed as

$$\begin{aligned}&H(z_i,J_{z_i},u_i,u_j,\varphi_i,\varphi_j)=\\&\quad\mathcal{Y}_1(x)+\mathcal{Y}_2(x)+\mathcal{Y}_3(x)+V_{z_i}^{\mathrm{T}}\dot{z}_i\end{aligned} \tag{9}$$

where $J_{z_i}=\dfrac{\partial J_i(z_i)}{\partial z_i}$ and $J_{z_i}(0)=0$. Because of the Bellman's optimality principle, we can derive the following CHJB equation

$$\min_{u_i,w_i}H(z_i,J^*_{z_i},u_i,u_j,\varphi_i,\varphi_j)=0 \tag{10}$$

where $J^*_{z_i}$ denotes the local optimal value function with $J^*_{z_i}(0)=0$. Therefore, the optimal control policies are given by

$$\begin{aligned}&u_i=-\frac{1}{2}(b_i+d_i)R_{ii}^{-1}f_i^{k\mathrm{T}}(x_i)J^*_{z_i},\\&\varphi_i=-\frac{1}{2\sigma_2}(b_i+d_i)P_{ii}^{-1}\mu_i^{\mathrm{T}}(x_i)J^*_{z_i}\end{aligned} \tag{11}$$

III. Event-Based Robust Optimal Consensus Control

In this section, we present a distributed ISMC framework by introducing a novel measurement error term, which guarantees the finite-time convergence of consensus errors to the origin.

A. Finite-Time ISMC

To handle the matched disturbances, we propose a distributed finite-time ISMC scheme. Accordingly, we have

$$u_i(t)=u_M(t)+u_{UN}(t) \tag{12}$$

Therefore, the integral SMS is described as

$$\begin{aligned}\mathcal{S}_i(t)=&\,x_i(t)-x_0(t)-(x_i(0)-x_0(0))-\\&\int_0^t[g^k(x_i)-g^k(x_0)+f_i^k(x_i)u_{UN}+\\&\mu_i(x_i)\sigma_i(t)]\mathrm{d}t\end{aligned} \tag{13}$$

Furthermore, one has

$$\dot{\mathcal{S}}_i(t)=f_i^k(x_i)(u_M(t)+\eta_i(t)) \tag{14}$$

To achieve optimal consensus and accelerate the convergence rate, we develop an ISMC algorithm as

$$\begin{aligned}u_i(t)=&\,u_{UN}(t)-p_1(f_i^k(x_i)^{\mathrm{T}}\mathcal{S}_i(t))^2-\\&p_2\mathrm{sign}(f_i^k(x_i)^{\mathrm{T}}\mathcal{S}_i(t))\end{aligned} \tag{15}$$

where p_1 and p_2 are the gains satisfying $p_1,\ p_2>0$.

To reduce the communication resource consumption between plant and controller while conserving computational resources, we propose an ETC for the dynamic system in Eq. (3) governing the local neighbor consensus tracking error. Consequently, we derive

$$E_k(t)=\begin{cases}\hat{z}_i-z_i(t),\ t\in(t_k^i,t_{k+1}^i);\\0,\ t=t_k^i\end{cases} \tag{16}$$

where $z_i(t)$ is the current state and $\hat{z}_i=z_i(t_k^i)$ is the sampled state.

Considering the event-triggered scenario, we design a consensus controller based on event-triggered ISMC.

$$\begin{aligned}u_i(t_k^i)=&\,u_{UN}(t_k^i)-p_1(f_i^k(x_i(t_k^i))^{\mathrm{T}}\mathcal{S}_i(t_k^i))^2-\\&p_2\mathrm{sign}(f_i^k(x_i(t_k^i))^{\mathrm{T}}\mathcal{S}_i(t_k^i))\end{aligned} \tag{17}$$

Therefore, the measurement error is given as

$$\begin{aligned}E_k(t)=&\,u_{UN}(t_k^i)-u_i(t)=\\&u_{UN}(t_k^i)-p_1(f_i^k(x_i(t_k^i))^{\mathrm{T}}\mathcal{S}_i(t_k^i))^2-\\&p_2\mathrm{sign}(f_i^k(x_i(t_k^i))^{\mathrm{T}}\mathcal{S}_i(t_k^i))-\\&u_i(t)+p_1(f_i^k(x_i)^{\mathrm{T}}\mathcal{S}_i(t))^2+\\&p_2\mathrm{sign}(f_i^k(x_i)^{\mathrm{T}}\mathcal{S}_i(t))\end{aligned} \tag{18}$$

Furthermore, the event-triggered optimal control policies are given by

$$u^*_{UN}=-\frac{1}{2}(b_i+d_i)R_{ii}^{-1}f_i^{k\mathrm{T}}(x_i)J^*_{z_i} \tag{19}$$

$$\varphi^*_i=-\frac{1}{2\sigma_2}(b_i+d_i)P_{ii}^{-1}\mu_i^{\mathrm{T}}(x_i)J^*_{z_i} \tag{20}$$

Remark 1 In the event-triggered mechanism, the initiation condition is jointly determined by the error margin of the event trigger and the threshold related to the state. When the event is triggered at time t_k, the error $e_k(t)$ is effectively reset to zero. Meanwhile, the controller receives an update based on this new measurement and maintains its constancy within $\forall t\in[t_k^i,t_{k+1}^i)$ by using zero-order hold (ZOH).

Assumption 3 The controllers u^*_{UN} exhibit Lipschitz continuity, which means that there is a real constant $l_e>0$ such that

$$\|u^*_{UN}(z_i)-u^*_{UN}(z_i(t_k^i))\|\leqslant l_e\|E_k\| \tag{21}$$

Assumption 4 The policy φ^*_i satisfies the following condition

$$2\sigma_2\varphi_{\max}(P)\|\varphi^*_i\|^2\leqslant\varphi_{\min}(Q)\|z_i\|^2 \tag{22}$$

where $\varphi_{\min}(\cdot)$ and $\varphi_{\max}(\cdot)$ represent the smallest and biggest singular values, respectively.

Remark 2 Assumption 3 is for revealing the relationship between the optimal control policies $u^*_{UN}(z_i)$ and $u^*_{UN}(z_i(t^i_k))$. It is satisfied in many applications, in which the controller is affine with respect to E_k. Moreover, Assumption 4 represents a standard condition that facilitates the derivation of the trigger condition and can be validated through numerical simulation results, which are also incorporated in Ref. [27].

B. Optimal Consensus Control Design

As is commonly understood, in order to approach the local optimal value functions J^*_i and $J^*_{z_i}$, only a single critic NN structure is presented in the the following paper. Consequently, we are able to obtain

$$J^*_i(z_i) = \vartheta^{\mathrm{T}}_i \phi_i(z_i) + \zeta_i(z_i) \tag{23}$$

where ϑ^{T}_i is denoted as the critic NN ideal weights, and $\phi_i(z_i)$ represents the activation function. The critic NN approximation error is expressed as $\zeta_i(z_i)$.

Therefore, the control policies for agent i are approximated with

$$\hat{u}_{UN} = -\frac{1}{2}((b_i+d_i)R^{-1}_{ii} f^{k\mathrm{T}}_i(x_i))\nabla\phi^{\mathrm{T}}_i\hat{\vartheta}_i \tag{24}$$

$$\hat{\varphi}_i = -\frac{1}{2\sigma_2}(b_i+d_i)P^{-1}_{ii}\mu^{\mathrm{T}}_i(x_i)\nabla\phi^{\mathrm{T}}_i\hat{\vartheta}_i \tag{25}$$

The goal of the critic NN is to minimize the error function E_i of agent i, then one has

$$E_i = \frac{1}{2}e_i(\hat{\vartheta}_i)^{\mathrm{T}} e_i(\hat{\vartheta}_i) \tag{26}$$

By differentiating E_i, we derive

$$\dot{\hat{\vartheta}}_i = -\wp_T\frac{\partial E_i}{\partial\hat{\vartheta}} = -\wp_T\nabla\phi^{\mathrm{T}}_i(z_i)\dot{z}_i e_i(\hat{\vartheta}) \tag{27}$$

where $\wp_T > 0$.

This paper adopts the method of concurrent learning to learn the weights of NNs by relaxing the requirements for PE conditions. By utilizing the concurrent learning method, one has

$$E_{\mathrm{coi}} = \frac{1}{2}\sum_{p=1}^{s}(e_{i,p}(\hat{\vartheta}_i)^{\mathrm{T}})e_{i,p}(\hat{\vartheta}_i) \tag{28}$$

In order to circumvent the need for an initial admissible control, the Lyapunov function J_{uni} is introduced as

$$\dot{J}_{\mathrm{uni}} = \nabla V^{\mathrm{T}}_{\mathrm{uni}}\dot{z}_i < 0 \tag{29}$$

Assuming a positive definite matrix $\Lambda_i \in \mathbb{R}^{n\times n}$ exists, we derive

$$\begin{aligned}\nabla V^{\mathrm{T}}_{\mathrm{uni}}\dot{z}_i = \nabla V^{\mathrm{T}}_{\mathrm{uni}}\Lambda_i\nabla V_{\mathrm{uni}} \leqslant\\ -\lambda_{\min}(\Lambda_i)\|\nabla V_{\mathrm{uni}}\|^2\end{aligned} \tag{30}$$

Assumption 5 (1) The weights, activation functions, and approximate errors of critic NN are bounded and satisfy $\|\vartheta_i\| \leqslant \vartheta_K$, $\|\phi_i\| \leqslant \phi_K$, and $\|\zeta_i\| \leqslant \zeta_K$, where ϑ_K, ϕ_K, and ζ_K are positive constants. (2) The gradients of the activation functions and the approximation errors in critic NN are bounded and satisfy $\|\nabla\phi_i\| \leqslant \nabla\phi_K$ and $\|\nabla\zeta_i\| \leqslant \nabla\zeta_K$, where $\nabla\zeta_i = \dfrac{\partial\zeta_i(z_i)}{\partial z_i}$, $\nabla\phi_K$ and $\nabla\zeta_i$ are positive constants.

Assumption 6 Suppose the gradient of the activation function $\phi_i(z_i)$ of the critic NN is Lipschitz continuous, that is

$$\|\phi_i(z_i) - \phi_i(\hat{z}_i)\| \leqslant L_u\|E_k\| \tag{31}$$

where L_u is a positive real constant.

Ensure that the MASs in Eq. (1) incorporating optimal local consensus control policies satisfy ETC design. Consequently, the closed-loop system can achieve asymptotically stable, and it is guaranteed that the error in critic weight estimation remains uniform ultimate boundedness (UUB).

Thus, we choose the Lyapunov function as

$$\begin{aligned}L_i = J^*_i(z_i) + J^*_i(z_i(t^i_k)) + \frac{1}{2}\tilde{\vartheta}^{\mathrm{T}}_i\tilde{\vartheta}_i + \wp_{\mathrm{un}}V^*_{\mathrm{uni}}(z_i) =\\ \mathcal{J}_1+\mathcal{J}_2+\mathcal{J}_3+\mathcal{J}_4\end{aligned} \tag{32}$$

where $\tilde{\vartheta}_i = \vartheta_i - \hat{\vartheta}_i$.

Case 1: Upon event triggering, $t\in(t^i_k, t^i_{k+1})$, $j^*_i(z_i)$ can be converted to

$$\begin{aligned}\dot{\mathcal{J}}_1 \leqslant & -z^{\mathrm{T}}_i Q_{ii}z_i - \sigma_2\varphi^{\mathrm{T}}_{i\varphi}P_{ii}\varphi_{i\varphi} - \sigma_2\hat{\varphi}^{\mathrm{T}}_i P_{ii}\hat{\varphi}_i -\\ & \hat{u}^{\mathrm{T}}_{UN}(\hat{z}_i)R_{ii}\hat{u}_{UN}(\hat{z}_i) - \sum_{j\in N_i}(u^{*\mathrm{T}}_{UN}R_{ij}u^*_{UN}) +\\ & \Gamma^2_{f_i}L^2_u\|E_k(t)\|^2\|\hat{\vartheta}_i\|^2 + \Gamma_{f_i\phi_K}\|\hat{\vartheta}_i\|^2 +\\ & \Gamma_{f_i\phi_K}\|\tilde{\vartheta}_i\|^2 + \Gamma_{f_i\zeta_K} + \Gamma_{\mu_i\phi_K}\|\tilde{\vartheta}_i\|^2 + \Gamma_{\mu_i\zeta_K}\end{aligned} \tag{33}$$

where $\Gamma_{f_i\phi_K} = \dfrac{(b_i+d_i)^2}{2\|R_{ii}\|}\Gamma^2_{f_i}\phi^2_K$, $\Gamma_{f_i\zeta_K} = \dfrac{(b_i+d_i)^2}{2\|R_{ii}\|}\Gamma^2_{f_i}\zeta^2_K$, $\Gamma_{\mu_i\phi_K} = \dfrac{(b_i+d_i)^2}{2\sigma_2\|P_{ii}\|}\Gamma^2_{\mu_i}\phi^2_K$, and $\Gamma_{\mu_i\zeta_K} = \dfrac{(b_i+d_i)^2}{2\sigma_2\|P_{ii}\|}\Gamma^2_{\mu_i}\zeta^2_K$.

Since the event has not been triggered, we have

$$\dot{\mathcal{J}}_2 = \dot{V}^*_i(z_i(t_k)) = 0 \tag{34}$$

Hence, it yields that

$$\begin{aligned}& \dot{\mathcal{J}}_3 + \dot{\mathcal{J}}_4 \leqslant\\ & -\frac{1}{2}\wp_{\mathrm{T}}\lambda_{\min}(\pi_{i\mathrm{T}}\pi^{\mathrm{T}}_{iT} + \sum_{p=1}^{s}\pi_{iT,p}\pi^{\mathrm{T}}_{iT,p})\|\tilde{\vartheta}^2_i\| +\\ & \frac{1}{2}\wp_T(s+1)\zeta^2_K + \frac{\wp_{unT}\Gamma^2_{i0}}{16\lambda_{\min}(\Lambda_i)}\end{aligned} \tag{35}$$

where $\pi_{iT} = \dfrac{\xi_{iT}}{\left(1+\xi^{\mathrm{T}}_{iT}\xi_{iT}\right)}^2$, $\pi_{iT,p} = \dfrac{\xi_{iT,p}}{\left(1+\xi^{\mathrm{T}}_{iT,p}\xi_{iT,p}\right)^2}$, and $\Gamma_{i0} = \|d_i+b_i\|\zeta_K\|R_{ii}\|\Gamma^2_{f_i} + \left(\dfrac{1}{\sigma_2}\right)\|d_i+b_i\|\zeta_K\|P_{ii}\|\Gamma^2_{\mu_i}$.

Combining Formulas (33)–(35), we can conclude $\dot{L}_i \leqslant 0$.

Case 2: At the moment that the event is triggered, $t = t^i_k$, based on Eq. (32), the Lyapunov function difference is expressed as

$$\Delta L_i = \Delta\mathcal{J}_1(t) + \Delta\mathcal{J}_2(t) + \Delta\mathcal{J}_3(t) + \Delta\mathcal{J}_4(t) \tag{36}$$

Therefore, we have

$$\Delta\mathcal{J}_{1,3,4}(t)=\mathcal{J}_1(\lim_{T\to 0}(t+T))-\mathcal{J}_1(t)\leqslant 0 \tag{37}$$

Furthermore, as for $\Delta\mathcal{V}_2(t)$, we obtain

$$\Delta\mathcal{J}_2(t)=\mathcal{J}_2(t^i_{k+1})-\mathcal{J}_2(t^i_k)\leqslant -k(t^i_{k+1}-t^i_k) \tag{38}$$

where $k(\cdot)$ is the class-k function. By integrating these two parts, it can be concluded that the nonlinear MASs in Eq. (1) are asymptotically stable, and the weight estimation error is UUB.

IV. Simulation Study

A practical case to verify the effectiveness of the aforementioned control is presented in this section. Figure 1 shows a directed graph. In Fig. 1, 1 represents the leader, and the rest respectively represent three followers. From this, it can be known that the message of the leader can only be passed on to the follower 3.

Therefore, the system dynamics is described by

$$\begin{cases}\dot{x}_{i,1}=g^{\kappa(t)}_{i,1}(x_i)+f^{\kappa(t)}_{i,1}(x_i)+\varphi_i,\\ \dot{x}_{i,2}=g^{\kappa(t)}_{i,2}(x_i)+f^{\kappa(t)}_{i,2}(x_i)u_i(t)\end{cases} \tag{39}$$

where the external disturbance $\varphi_i=a_{i1}x_{i1}\sin(a_{i2}x_{i2})$ with $a_{i1}\in[-2,2]$, $a_{i2}\in[-2,2]$, and $i=1,2,3$.

In this simulation, the initial conditions are taken as $z_1=[-3;6]^{\mathrm{T}}$, $z_2=[-2;2]^{\mathrm{T}}$, $z_3=[-1;3]^{\mathrm{T}}$, $\omega_i=[0;0;0]^{\mathrm{T}}$, $i=1,2,3$, $x_1=[4;2]^{\mathrm{T}}$, $x_2=[3;4]^{\mathrm{T}}$, $x_3=[6;2]^{\mathrm{T}}$, and $x_0=[-1;1]^{\mathrm{T}}$. For $i=1,2,3$, let $Q_{ii}=I_2$, $R_{ii}=I_2$, and $T_{ii}=I_2$.

The traditional SMC sliding-mode function is designed as

$$S_i=k_i(x_{i1}-x_{01})+x_{i2}-x_{02} \tag{40}$$

where $k_1=[-0.5,0]^{\mathrm{T}}$, $k_2=[0.3,0]^{\mathrm{T}}$, and $k_3=[0.8,0]^{\mathrm{T}}$.

Figure 2 depicts the sliding-mode surfaces of traditional SMC. It is evident that, in contrast to the finite-time ISMC, the traditional SMC includes an arrival phase, and its initial position does not lie on the SMS. From Fig. 3, it can be observed that the consensus errors converge to the SMS $S_i(t)=0$, $i=1,2,3$, at the initial time and subsequently remain on the sliding surface, indicating that the settling time satisfies $T_r\leqslant T_{r\max}$. Consequently, it can be concluded that the event-triggered control law based on sliding-mode ensures the finite-time reachability of the SMS. In addition, Fig. 4 depicts the weight curves of NN. It is observed that the weights eventually converge to $\omega_1=[0.0100,-0.0478,0.1603]^{\mathrm{T}}$, $\omega_2=[0.0005,-0.0063,0.0410]^{\mathrm{T}}$, and $\omega_3=[0.0025,-0.0130,0.0705]^{\mathrm{T}}$.

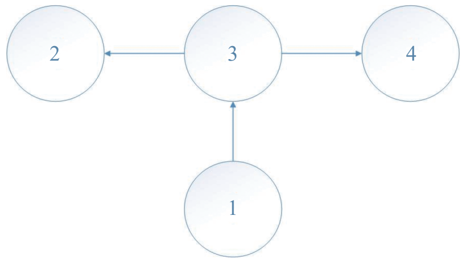

Figure 1 Leader-follower communication topology.

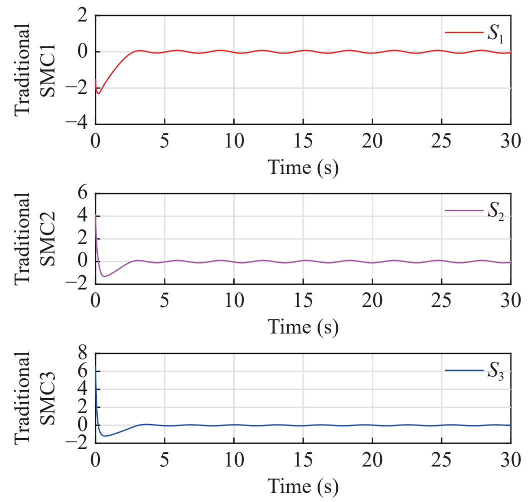

Figure 2 Sliding-mode surface of traditional SMC.

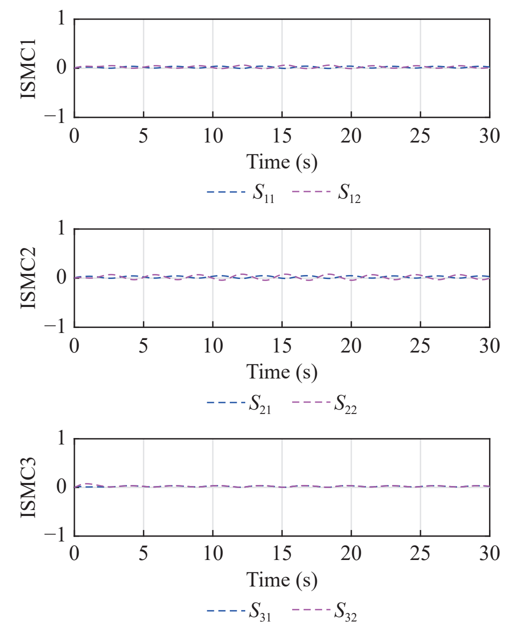

Figure 3 Sliding-mode surface of ISMC.

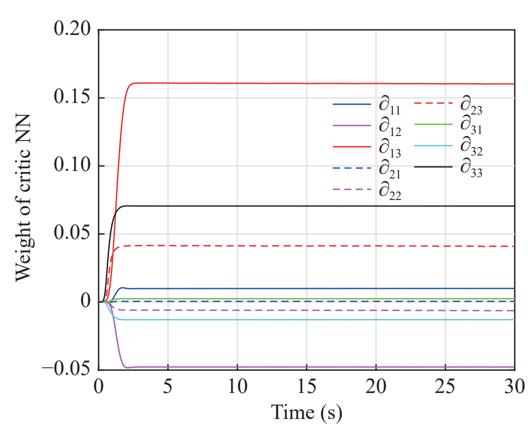

Figure 4 Weight convergence of critic network.

Finally, according to Table 1, it is shown that the ISMC+ADP controller can produce a better convergence rate than that of the SMC controller. To verify the disturbance rejection ability of the proposed method, the external disturbance $\varphi_i = a_{i1}x_{i1}\sin(a_{i2}x_{i2})$ with $a_{i1} \in [-2,2]$, $a_{i2} \in [-2,2]$, and $i = 1,2,3$.

Table 1 Performance comparison.

Index	SMC	ISMC+ADP
Convergence rate	Fast (3 s)	Fastest (1–2 s)
Anti-interference ability	Strong	Strongest

V. Conclusion

In this article, an ADP-based robust optimal consensus control method is proposed for switched nonlinear MASs using a finite-time ISMC. First, we reformulate the unmatched uncertainty robust control problem as an optimal control problem using an auxiliary system. In addition, the event-triggered ISMC eliminates the matched disturbances and the computing and communication resources are conserved. By developing a critic NN to approximate the optimal value function, the proposed concurrent learning method eliminates the need for an initial admissible control and PE conditions. Finally, the weight estimation error of NN is UUB. In the framework of this work, the inverse optimal control of nonlinear MASs is also a problem worthy of further exploration.

VI. Acknowledgment

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